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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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VOLUME LV | JANUARY 1922 | NUMBER 1

COLOR-TRANSPARENCY OF RAY-FILTERS IN USE AT YERKES OBSERVATORY

By ERNEST C. BRYANT

ABSTRACT

Spectro-photometric transmission curves for two ray-filters in use at Yerkes Observatory, and for three colored glasses used in the Hess-Ives tint photometer are given for 400 to 750 $\mu\mu$. The filters investigated are J 1, M 1, December 1907, Beta 7 and Beta 10; the glasses are a red, a green and a blue-violet.

Lemon-Brace spectro-photometer.—Suggestions as to the adjustment and calibration of this instrument, which was used in obtaining the foregoing curves, are given, and also measurements of the *relative selective absorption of nicol prism and silver strip* from 475 to 750 $\mu\mu$.

Color-curve of the 40-inch objective at Yerkes Observatory is reproduced in Figure 5.

The 40-inch refractor at Yerkes Observatory, although designed only for visual observations, has been available for astronomical photography since 1900 by means of a method perfected by G. W. Ritchey and described by him in the *Astrophysical Journal* for December of that year. The method consists of the use of a ray-filter which absorbs those waves of short lengths for which the objective is uncorrected, combined with photographic plates which are especially sensitive to the yellow, a color which the ray-filter transmits freely, and which comes in the region for which the color-curve of the 40-inch objective is very nearly flat.

The filters first used by Ritchey were made by Carbutt, of Philadelphia. R. J. Wallace came to Yerkes in 1906 and immediately began fitting filters to the various telescopes, especially to

give photo-visual results in connection with the isochromatic plates in use, mainly the Cramer Isochromatic and Trichromatic.

After several trials the 8×10 filter marked "Dec. '07," consisting of a colored gelatine film between two sheets of optical glass, was put into regular use in the parallax program and other work with the 40-inch telescope. After some years the gelatine began to deteriorate at the edges of the film, and the filter was finally superseded by a similar one made by Mees, of the Eastman Research Laboratory. He planned to duplicate the former filter as closely as possible. Of the two which he made, that called "M 1" was put into use in the parallax work in May 1920.

Wallace made similar filters for the 2-foot reflector and the 6-inch ultra-violet camera. On the reflector, filter "Beta 7" 3×3 inches in size, was used from July 1906 until it was accidentally broken in July 1917. On the camera, "Beta 10," 4×5 inches, has been in use from February 1907 till the present time. Both have been shown to give substantially correct photo-visual magnitudes when used with Cramer Isochromatic plates. By comparison with photographic magnitudes found with ordinary plates on the same instruments, color-indices are obtained and spectral types inferred (see *Astrophysical Journal*, **27**, 169, April 1908).

After the accidental breaking of "Beta 7," several others were tried, both Wallace and Eastman; but none was satisfactory. In 1917 Petitdidier furnished two tinted Jena glass filters, No. F 4351, one 3×3 inches which replaced "Beta 7"; and one 6×6 inches. The former, called "J 1," has been used on the reflector since 1917.

Photographic determinations of the spectral intensity-curves for Seed 27 plates without ray-filter and for Cramer Trichromatic plates with ray-filter have been made, but no direct measurements of the color transmission of the filters themselves have previously been obtained. The loan of a Lemon-Brace Polarization Spectrophotometer from the Ryerson Physical Laboratory has enabled these determinations to be made this summer, and the following results have been obtained. This instrument has been described in this *Journal*, January 1900, September 1902, and April 1914, but a

brief statement of the method used by the author for adjusting and calibrating it may be helpful to those having their first experience with the process.

Referring to Figure 1, the instrument when in adjustment must fulfil the following conditions: (1) slits S_1, S_2, and S_3 must be at the principal foci of their respective objectives; (2) optical axes of collimators C_1 and C_2 and of the telescope T must be in the same plane, which must be perpendicular to the refracting edges of the Brace prism AEI; (3) light from S_1 must strike prism at B, making $AB = 1/4AE$, at the angle of minimum deviation for the Fraunhofer lines D, emerging at F, making $IF = 1/4IE$; (4) light from S_2 similarly must strike the prism at D, making $ED = 1/4EA$, at the angle of minimum deviation for the D lines. It must meet the division plane CI at H, the same place where the light from S_1 meets it, and that part reflected by the silver strip on CI must follow the same direction HF, FJ as the light from S_1. In the instrument used, the Brace prism merely rested on its platform with nothing to locate its position on the platform or its orientation.

After making adjustments (1) and (2), beams of sunlight were reflected into S_1 and S_2, directed so as to emerge through the centers of their objectives. This latter condition was carefully maintained throughout the process. The prism was oriented for minimum deviation of D lines, the slits were opened and coincidence of emerging beams at F was tested by placing a piece of white paper against the face of the prism at that point. The prism was moved parallel to line S_1B, retaining minimum deviation of D lines, until the beam from S_2 reflected at H emerged at F exactly between the beams from S_1 which passed above and below the silver strip at H. Coincidence in direction of emergent beams was tested by receiving them on the white paper held in front of the telescope objective and then beyond the eyepiece J. If not coincident in direction, collimator C_2 must be swung on its axis to obtain coincidence. This will require a repetition of the preceding adjustment. These two adjustments must be thus repeated until the three beams emerge at F in the same vertical line, enter the telescope and emerge at the eyepiece likewise in the

same vertical line. The final adjustment of C_2 is made by narrowing S_1 and S_2 and testing for coincidence of Fraunhofer lines. The lines given by light through S_2 should be exact continuations of the lines given by light through S_1. For this test the "*b*" lines were found most favorable. If the slight motion of C_2 to secure this last step displaces the central emergent beam at *F* the prism must be moved enough to restore it and the last step repeated. When the adjustments are finally secured, collimator C_2 is clamped, and

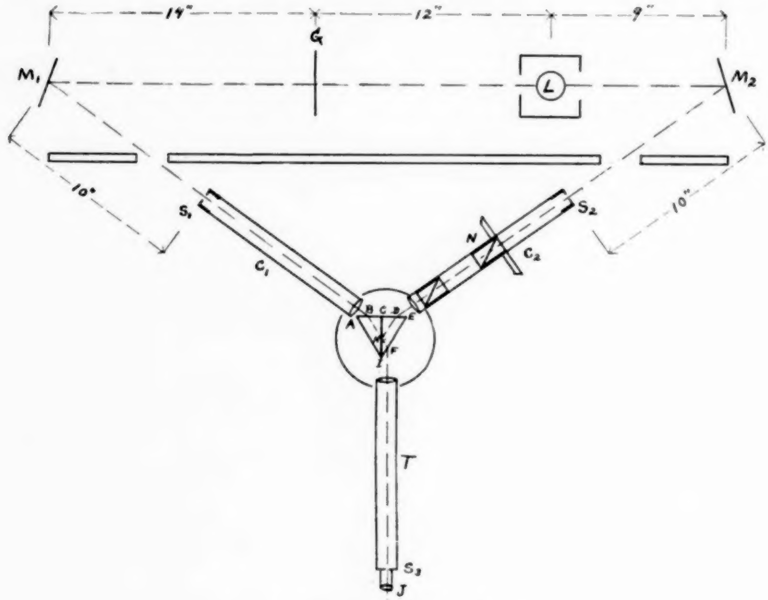

FIG. 1.—Lemon-Brace spectro-photometer

prism *AEI* is secured in its position. The instrument is then ready for calibration. The author did this by using the Fraunhofer lines of the solar spectrum, and determining the reading of the scale attached to the telescope when sighted on each of nineteen lines.

After calibration, the instrument was set up as shown in Figure 1. *L* is a 50-watt, nitrogen-filled, electric lamp of white glass. It is inclosed in a box with openings as shown. M_1 and M_2 are mirrors adjustable about vertical and horizontal axes so as to reflect the light from *L* along the optical axes of C_1 and C_2.

In order that the illumination of the silver strip by M_2 might be brighter than that of the background by M_1 when the nicols were parallel, it was necessary to have L much nearer M_2 than M_1 and also to place a piece of ground glass, G, in position as shown. An improvement could be made by the use of a lamp of higher power with an adjustable rheostat. This would give better illumination at the ends of the spectrum and also enable one to reduce the brightness at the middle of the spectrum. With the present arrangement the ends are too faint for accurate measurement, while the middle is too bright for comfort.

While making measurements, the room was darkened and the lamp, mirror, etc., were covered with black cloth. Slits S_1 and S_2 were opened to a width of $\frac{1}{2}$ mm. Eyepiece J was removed, and slit S_3 made as narrow as possible without producing diffraction fringes. The telescope was set at such a scale-reading as to bring light of the desired wave-length to S_3. Without having the ray-filter in place, five settings of nicol N were made, balancing the illumination of the silver strip against its background. The average $\sin^2$ of these readings gave the fractional part of the light of the chosen wave-length entering S_2 necessary to balance that entering S_1. The ray-filter was then put in front of S_1 and five other settings of N were made. The average sine^2 of these readings gave the fractional part of the light entering S_2 now necessary to balance that transmitted by the ray-filter. The ratio between the two averages gave the fractional part of the incident light of the chosen wave-length which was transmitted by the ray-filter. By making the measurements in this way, the percentage of transmission at any particular wave-length could be obtained at any time with no necessity of taking any other measurements in the series.

The current through the lamp L sometimes dropped to 50 per cent, or less, of its normal value, when the motor raising the floor in the big dome was started, but no perceptible effect on the balancing of illuminations in the Brace prism was produced thereby.

The five ray-filters already described were examined for their color-transmission and the percentages of transmission of the selected wave-lengths are given in the following table. The curves for filters "Dec. '07" and "M 1" appear in Figure 2. It will be

seen how closely the filters "Dec. '07" and "M 1" correspond in their transmission to the flat part of the color-curve of the 40-inch objective shown in Figure 3, which is taken from the investigation published by Philip Fox in this *Journal*, **27**, 237, May 1908. The fact that the filters transmit the longer wave-lengths for which the color-curve slopes upward is neutralized by the lack of sensitiveness of the photographic plates to those wave-lengths. The

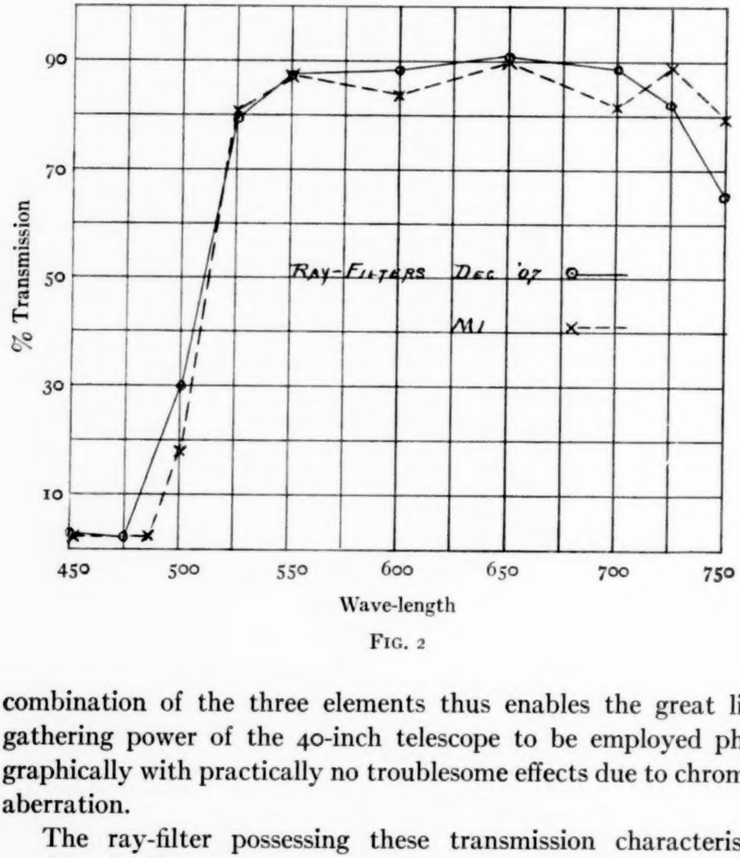

FIG. 2

combination of the three elements thus enables the great light-gathering power of the 40-inch telescope to be employed photographically with practically no troublesome effects due to chromatic aberration.

The ray-filter possessing these transmission characteristics, combined with a plate having a maximum sensitiveness to the yellow, gives a combination whose position of maximum sensitiveness corresponds to that of the retina of the human eye. This is one of the reasons why the determinations of stellar magnitudes

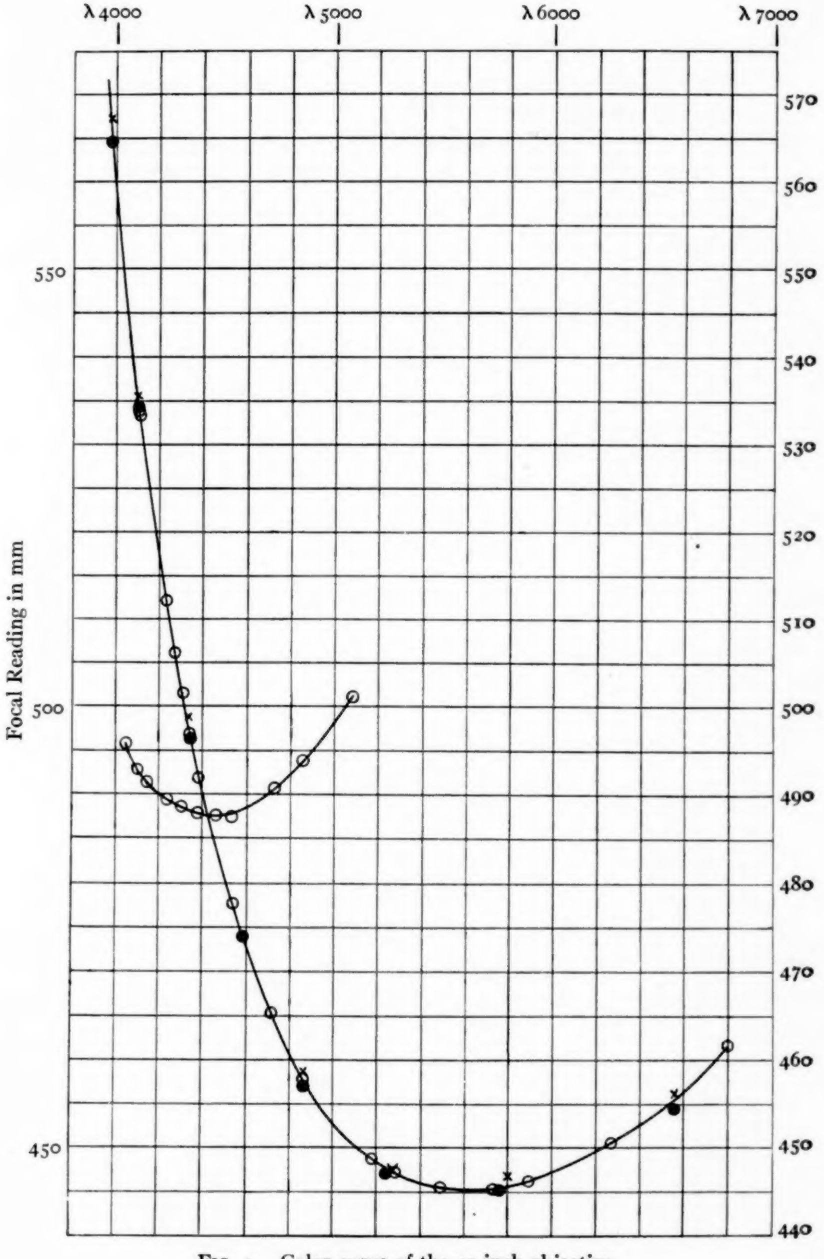

FIG. 3.—Color-curve of the 40-inch objective

made by Professor J. A. Parkhurst by his photographic method have such close correspondence to the visual magnitudes.

TABLE I

TRANSMISSION-PERCENTAGES OF RAY-FILTERS

Wave-Length	J 1	M 1	Beta 10	Dec. '07	Beta 7
750	87.1	79.4	82.0	64.8	72.2
725	85.5	89.2	73.9	81.8	79.3
700	89.3	81.8	74.0	88.7	78.0
650	92.6	90.3	82.2	91.0	87.1
600	87.7	84.0	86.2	88.5	83.5
550	88.7	87.8	88.8	87.6	87.8
525	87.3	81.1	85.3	79.6	81.5
500	57.6	18.5	40.0	29.8	40.8
486	7.9	2.0	14.4	....	....
475	2.5	0.6	6.0	2.0	12.5
450	....	....	....	2.3	4.9

The three colored glasses in use with a Hess-Ives Tint Photometer were also examined for their color-transparency. The results appear in the following table and in Figure 4.

TABLE II

TRANSMISSION-PERCENTAGES OF COLORED GLASSES, HESS-IVES TINT PHOTOMETER

Red Glass		Green Glass		Blue-Violet Glass	
Wave-Length	Percentage	Wave-Length	Percentage	Wave-Length	Percentage
766	0.0	628	0.0	495	0.0
750	11.7	610	0.04	486	0.8
725	57.7	590	1.1	475	3.9
700	73.0	570	6.0	465	9.2
675	78.7	550	15.9	455	14.4
650	78.4	530	19.5	445	19.1
625	65.0	510	10.5	435	20.9
600	3.4	490	0.8	425	18.7
489	0.0	468	0.0	415	0.0

It will be seen that the red glass has a maximum transmission of 78.7 per cent for light of wave-length 675 $\mu\mu$. Its average transmission in the region for which it transmits appreciably, i.e., between 766 $\mu\mu$ and 589 $\mu\mu$ is 51.5 per cent of the incident light. In the same way the maximum transmission for the green glass

is 19.5 per cent for light of wave-length 530 $\mu\mu$, and its average transmission for the region between 628 and 468 $\mu\mu$ is 6.7 per cent. The blue-violet glass has a maximum transmission of 20.9 per cent for light of wave-length 435 $\mu\mu$, and an average transmission for the region between 495 $\mu\mu$ and 415 $\mu\mu$ of 10.9 per cent. It is very noticeable how little overlapping there is in the light transmitted by either pair of adjacent glasses.

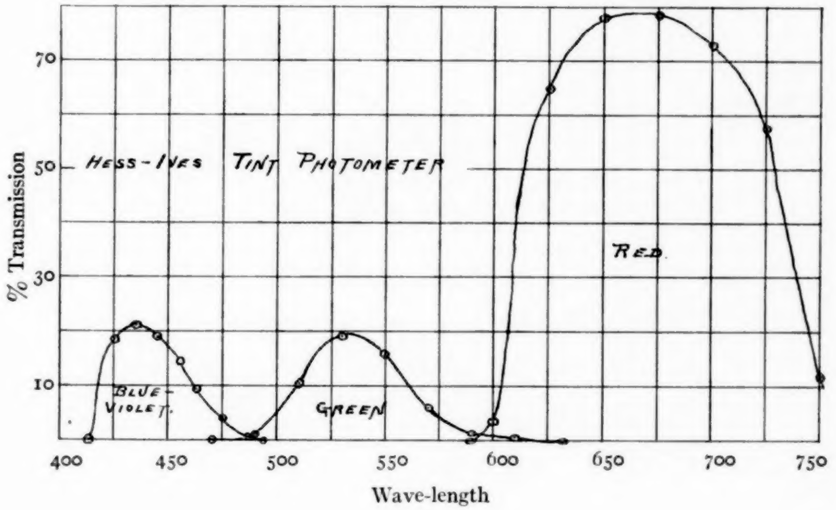

FIG. 4

An interesting piece of incidental information is the relative selective absorption exerted by the nicol prisms and the silver strip. When the readings are taken with no ray-filter in front of S_1, the angle through which the nicol N must be rotated to produce a balancing of illuminations is not the same for all wave-lengths. At first it increases as wave-length decreases, remains practically uniform from $\lambda = 600\ \mu\mu$ to $\lambda = 525\ \mu\mu$, and then decreases with the wave-length, showing that there is selective absorption corresponding in relative values to the $\sin^2$ of the angle of rotation of the nicol. The table below is computed from the average values obtained in the measurements of filters "M 1," "Beta 10," and "J 1."

The measurements on ray-filters "Dec. '07" and "Beta 7" were made after the instrument had been taken down, readjusted

TABLE III

Wave-length	750	700	650	600	550	525	500	485	475
$\sin^2$	0.246	.415	.563	.613	.610	.611	.575	.535	.513

and calibrated, and therefore could not be included. Figure 5 gives the curve corresponding to this table.

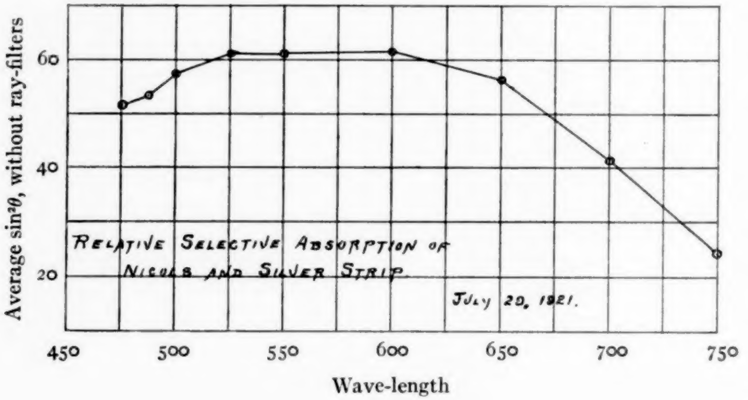

FIG. 5

The author is greatly indebted to Professor J. A. Parkhurst for suggesting the investigation and for his kind assistance during its progress and in the preparation of this report.

MIDDLEBURY COLLEGE
October 1, 1921

SYSTEMATIC CORRECTIONS TO SPECTROSCOPIC AND TRIGONOMETRIC PARALLAXES[1]

By GUSTAF STRÖMBERG

ABSTRACT

Systematic corrections to reduce spectroscopic and trigonometric parallaxes to absolute parallaxes.—The fact that the systematic and accidental errors of spectroscopic parallaxes are proportional to the parallaxes themselves, while the errors in trigonometric parallaxes are independent of the values of the parallaxes, makes it possible, theoretically at least, to determine from a comparison of the two systems the true systematic corrections for each system and even for each observer. The basis of grouping the stars must be independent of each system and therefore Kapteyn's mean parallaxes, derived from apparent magnitudes and proper motions, were used. The factorial correction (x) for the spectroscopic parallaxes and the additive correction (s) for the trigonometric parallaxes, connected by the fundamental equation $x\pi_s=\pi_t+s$, were thus determined. For dwarfs x ranges from 0.97 to 1.07 for different spectral types; for giants the range is from 0.89 to 1.13, the extreme values being rather uncertain. The corresponding systematic correction to absolute magnitudes averages only about 0.1 for dwarfs, but may be somewhat larger for the giants. *For the separate observatories*, McCormick, Allegheny, Yerkes, and Mount Wilson, the reductions from relative to absolute parallaxes come out, respectively, $+0\overset{''}{.}0053$, $+0\overset{''}{.}0076$, $+0\overset{''}{.}0084$ and $-0\overset{''}{.}0010$ within about $\pm 0\overset{''}{.}002$. These values agree well with those obtained by van Maanen and Miss Wolfe. Hence the list of 1646 spectroscopic parallaxes recently published from Mount Wilson probably gives the absolute parallaxes without any appreciable systematic error.

The large number of spectroscopic parallaxes now available enables us to determine fairly accurate values of the systematic corrections for both spectroscopic and trigonometric parallaxes. In the derivation of the reduction tables for converting line intensities into absolute magnitudes used in the recent list of 1646 parallaxes[2] no corrections were applied to the trigonometric parallaxes, but it was found that if the corrections deduced by van Maanen and Miss Wolfe[3] had been used, the effect due to these corrections would have been less than 0.1 magnitude. The method used by van Maanen and Miss Wolfe consisted in comparing the means of the trigonometric parallaxes found by different observers for groups of stars of nearly the same mean apparent magnitude and mean proper motion. Their corrections represent the deviations of the

[1] *Contributions from the Mount Wilson Observatory*, No. 220.

[2] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 1, 1921.

[3] *Mt. Wilson Contr.*, No. 189.

results by individual observers from the mean system defined by all the trigonometric parallaxes. Since the systematic errors of individual observers must compensate each other to a large extent in the mean system, the corrections of van Maanen and Miss Wolfe are presumably close approximations to the true systematic errors.

Other determinations of systematic corrections of trigonometric parallaxes have been made by Flint[1] and B. Boss,[2] the former reducing his corrections to a system based upon the Yale parallaxes, the latter comparing the spectroscopic parallaxes published in *Mt. Wilson Contribution*, No. 142 (*Astrophysical Journal*, **46**, 313, 1917) with the different series of trigonometric parallaxes.

The comparison of spectroscopic and trigonometric parallaxes enables us, theoretically at least, to determine the true systematic corrections for each observer and for the spectroscopic system as well. This depends upon the fact that the systematic and accidental errors of the spectroscopic parallaxes are proportional to the parallaxes themselves, while the errors in the trigonometric parallaxes are independent of the size of the parallax. Hence, for any series of trigonometric parallaxes, when compared with the spectroscopic system, we have, as shown more in detail below, an equation of the form

$$x\pi_s = \pi_t + s,$$

from which the systematic correction s, and the correction factor x for the spectroscopic parallaxes, can be determined. The successful use of the equation of course presupposes a considerable range in the values of the parallaxes compared. Another important advantage of the method is that any error in the assumed values of the mean parallax of the comparison stars used for the reduction of the relative trigonometric parallaxes to absolute values goes over into s and is determined as a part of the systematic correction. In principle, therefore, the comparison permits us to establish an absolute system of parallaxes.

The principal object of this investigation being to determine systematic corrections of the spectroscopic parallaxes as a whole, taking into account the possibility of constant corrections to the trigono-

[1] *Astronomical Journal*, **29**, 189 (No. 696), 1916.

[2] *Ibid.*, **33**, 17 (No. 771), 1920.

metric parallaxes, only the larger lists of trigonometric parallaxes, viz., those derived at the McCormick, Allegheny, Yerkes, Mount Wilson, Sproul, and Yale observatories, have been discussed.

The following notation has been used:

M = Absolute magnitude as published in *Mt. Wilson Contribution*, No. 199.

ΔM = Systematic correction to M to obtain the most probable absolute magnitude.

T = Accidental error of $M+\Delta M$.

π = True parallax.

π_1 = Spectroscopic parallax based on the absolute magnitude M.

π_0 = Trigonometric parallax.

s = Systematic correction to π_0

ϵ = Accidental error of π_0+s.

We have then (cf. *Mt. Wilson Contribution*, No. 199, p. 4)

$$\left.\begin{array}{ll} & \pi=10^{0.2(\Delta M+T)}\pi_1=\pi_0+s+\epsilon \\ \text{or} & \sigma\tau\pi_1=\pi_0+s+\epsilon \\ \text{where} & \sigma=10^{0.2\Delta M}, \qquad \tau=10^{0.2T} \end{array}\right\} \tag{1}$$

Expanding τ in a power series,

$$\left.\begin{array}{l} \tau=1+E+\tfrac{1}{2}E^2=A+E \\ E=\dfrac{0.2T}{\text{Mod}} \\ A=1+\tfrac{1}{2}E^2 \end{array}\right\} \tag{2}$$

Thus

$$\sigma A\pi_1-s-\pi_0=\epsilon-E\sigma\pi_1 \tag{3}$$

As the right-hand side of this equation must be assumed equal to zero, we cannot group the stars according to the size of π_0 or π_1. If we group the stars according to values of π_0 the quantity $\bar{E}$ could be assumed zero,[1] but $\bar{\epsilon}$ would show a continuous decrease from positive to negative values as π_0 increases. The same holds for $\bar{E}$ if we group the stars according to π_1, and furthermore the weight of the unknown quantity σ would be systematically affected by the accidental errors in π_1. These circumstances correspond to

[1] A bar above a symbol denotes throughout this paper an algebraic mean.

the difference in slope of the two regression lines in the correlation theory. A neutral basis of grouping according to the size of the parallaxes must therefore be used, i.e., one for which we can assume the errors in the parallaxes to be independent of the errors in the measured quantities M and π_0. Such an independent basis of grouping is afforded by Kapteyn's mean parallaxes, which are a function of apparent magnitude and proper motion. For each star the mean parallax was therefore computed from Kapteyn's formula in *Groningen Publication*, No. 8, and groups of stars were then formed having mean parallaxes within certain limits. The mean spectroscopic and trigonometric parallax was then found for each group. Each of these groups then furnished an equation of condition.

From equation (3) we find

$$\sigma \bar{A} \bar{\pi}_1 - s - \bar{\pi}_0 = \bar{\epsilon} - \sigma \overline{E\pi_1} .$$

For a neutral basis of grouping we have $\bar{\epsilon}=0$ and $\bar{E}=0$. The second term on the right-hand side, however, is not quite zero, as E is to some extent dependent on π_1 and thus $\overline{E\pi_1}$ is not exactly equal to $\bar{E}\bar{\pi}_1$.

To reduce the right-hand side of the equation of condition to zero, we multiply equation (1) by

$$\frac{1}{\tau} = 1 - E + \tfrac{1}{2}E^2 = A - E$$

and find

$$\sigma\pi_1 = A(\pi_0 + s + \epsilon) - E(\pi_0 + s + \epsilon).$$

Thus

$$\frac{\sigma}{A}\pi_1 - s - \pi_0 = \epsilon\left(1 - \frac{E}{A}\right) - \frac{E}{A}(\pi_0 + s).$$

For a neutral basis of grouping, we have, including terms of second order of E,

$$\frac{\sigma}{\bar{A}}\bar{\pi}_1 - s - \bar{\pi}_0 = \bar{\epsilon}(1 - \bar{E}) - \bar{E}(\bar{\pi}_0 + s) = 0.$$

The equations of condition thus become

$$\left.\begin{aligned} x\pi_1 - s - \pi_0 &= 0 \\ \text{Weight} = \frac{1}{\bar{\epsilon}^2 + \overline{E}^2\,\bar{\pi}^2} &= \frac{N}{\overline{\epsilon}^2 + \overline{E}^2\,\bar{\pi}^2} \end{aligned}\right\} \qquad (4)$$

TABLE I

SYSTEMATIC CORRECTIONS—SEPARATE SOLUTION FOR EACH TYPE

Spectrum	M	x	McCormick s_1	Allegheny s_2	Yerkes s_3	Mount Wilson s_4	Sproul s_5	Yale s_6
A6 to F9....	1.0 to 5.5	1.056	+0.″0037 (53)	+0.″0020 (92)	+0.″0013 (25)	−0.″0009 (24)	−0.″0068 (35)	−0.″0008 (46)
		± .035	± .0020	± .0015	± .0019	± .0015	± .0021	± .0022
G0 to G9....	≦3.0	1.095	+0.0012 (35)	+0.0011 (69)	+0.0052 (10)	−0.0040 (13)	−0.0114 (11)	+0.0085 (11)
		± .134	± .0033	± .0031	± .0055	± .0036	± .0062	± .0082
G0 to G9....	≧3.1	0.975	−0.0055 (36)	+0.0038 (60)	−0.0048 (27)	+0.0221 (8)	−0.0130 (10)	−0.0008 (74)
		± .053	± .0044	± .0032	± .0034	± .0040	± .0054	± .0031
K0 to K9....	≦4.[illegible]	1.075	−0.0028 (25)	+0.0058 (54)	+0.0181 (7)	−0.0085 (26)	−0.0090 (8)	+0.0198 (7)
		± .113	± .0034	± .0025	± .0050	± .0017	± .0040	± .0056
K0 to K9....	≧4.1	1.030	+0.0040 (39)	+0.0069 (27)	+0.0109 (30)	−0.0012 (7)	+0.0133 (20)	+0.0054 (70)
		± .064	± .0061	± .0062	± .0052	± .0092	± .0067	± .0050
A6 to K9....	....	....	+ .0012 (188)	+0.0030 (302)	+0.0026 (99)	−0.0024 (78)	−0.0069 (84)	+0.0018 (208)
			± ″0016	± ″0011	± ″0014	± ″0010	± ″0016	± ″0016

where $x=\frac{\sigma}{A}$ is the quantity by which the spectroscopic parallaxes must be multiplied in order to obtain the best agreement with π_0+s. N is the number of stars in the group.

Assuming only that the quantities x and s are constant for a certain group of stars, we can determine them separately without introducing any limitation as to their size. The value of s found in this way is the systematic correction which reduces π_0 to *absolute* parallax.

We have assumed that the external accidental probable error in the trigonometric parallaxes determined at the McCormick, Allegheny, Yerkes, and Mount Wilson observatories is $\pm 0''010$. The probable errors of the spectroscopic parallaxes have been assumed to be $\pm 0.20\ \pi_1$. The weights of the equations of condition are therefore proportional to $N/(0.09\bar{\pi}^2+0.000225)$ where for $\bar{\pi}$ we have used $\frac{1}{2}(\bar{\pi}_1+\bar{\pi}_0)$, x being nearly equal to unity.

The following provisional reductions to absolute parallaxes were applied before the computations were made: For McCormick, Allegheny, and Yerkes observatories, $+0''005$; for Mount Wilson, $+0''002$; for Sproul and Yale the reductions were computed from the table given by Kapteyn in *Groningen Publications*, No. 24, page 15. The Sproul and Yale parallaxes were given half-weight.

The value of x was determined separately for giant and dwarf stars of different spectral types. For the first solution the systematic correction s was determined for each of the above-mentioned groups of trigonometric parallaxes. The results are given in Table I. The first column indicates the limits of spectral type, the second the division according to absolute magnitude. The probable errors for x and s are below the numbers to which they belong. The numbers of stars used are given in parentheses. The last line shows the weighted means of s for all types and absolute magnitudes.

Using these mean systematic corrections to the trigonometric parallaxes and recomputing the correction factor of the spectroscopic parallaxes, we find the values of x and ΔM_1, given in Table II. The quantity ΔM_1 is the correction that must be applied to M in

TABLE II

CORRECTION FACTORS FOR SPECTROSCOPIC PARALLAXES

Spectrum	M	x	ΔM_1
A6 to F9	1.0 to 5.5	1.059±.012	+0.12±0.02
G0 to G9	≦3.0	1.121±.040	+0.25±0.08
G0 to G9	≧3.1	0.999±.029	0.00±0.06
K0 to K9	≦4.0	1.001±.059	0.00±0.13
K0 to K9	≧4.1	0.960±.019	−0.09±0.04
F0 to F9	≦0.0	0.149±.103	+0.30±0.20
Ma to Md	<3.0	0.892±.081	−0.25±0.20
Ma to Md	>7.0	1.089±.086	+0.18±0.17

order to obtain the most probable parallax and is determined by the equation

$$\Delta M_1 = 5 \log x = \Delta M - 5 \log \bar{A}.$$

Another solution was made, using all the spectral types, A6 to Md, and solving all the equations as a simultaneous set of conditions, assuming the correction s to be constant for each observer. The results of this solution are given in Table III.

TABLE III

SIMULTANEOUS SOLUTION FOR ALL TYPES

	s		s
McCormick	−0.″0015±0.″0018	Mount Wilson	−0.″0038±0.″0014
Allegheny	+ .0010± .0015	Sproul	− .0085± .0025
Yerkes	+ .0019± .0021	Yale	+ .0004± .0021

Spectrum	M	x	ΔM_1
A6 to F	1.0 to 5.5	1.015±.040	+0.03±0.08
G	≦3.0	1.037±.078	+0.08± .16
G	≧3.1	0.969±.033	−0.07± .07
K	≦4.0	0.922±.074	−0.18± .17
K	≧4.1	0.940±.029	−0.14± .07
F	≦0.0	0.962±.247	−0.08± .57
M	<3.0	0.750±.154	−0.63± .45
M	>7.0	1.082±.045	+0.17± .09

From van Maanen's investigation we know, however, that the Sproul and Yale parallaxes have a considerable magnitude correction. Such an error would affect appreciably the determination of the factor x, and for this reason a final solution was made,

omitting the Sproul and Yale parallaxes. The results of this solution, which are given in Table IV, are probably the most trustworthy. They are in good agreement with those found by van Maanen and Miss Wolfe, which, reduced to the same system of corrections for reduction to absolute parallaxes as used here, are given in the last column of Table IV.

TABLE IV

SIMULTANEOUS SOLUTION (SPROUL AND YALE OMITTED)

	s	s (van Maanen)
McCormick	$+0''.0003 \pm 0''.0021$	$-0''.0016$
Allegheny	$+\ .0026 \pm\ .0017$	$+\ .0027$
Yerkes	$+\ .0034 \pm\ .0022$	$+\ .0036$
Mount Wilson	$-\ .0030 \pm\ .0014$	$-\ .0020$

Spectrum	M	x	ΔM_1
A6 to F	1.0 to 5.5	$1.041 \pm .045$	$+0.09 \pm 0.09$
G	$\leqq 3.0$	$1.135 \pm .092$	$+0.27 \pm\ .18$
G	$\geqq 3.1$	$0.987 \pm .037$	$-0.03 \pm\ .08$
K	$\leqq 4.0$	$1.012 \pm .086$	$+0.03 \pm\ .18$
K	$\geqq 4.1$	$0.972 \pm .034$	$-0.06 \pm\ .08$
F	$\leqq 0.0$	$1.090 \pm .254$	$+0.19 \pm\ .52$
M	< 3.0	$0.888 \pm .167$	$-0.26 \pm\ .41$
M	> 7.0	$1.068 \pm .052$	$+0.14 \pm\ .11$

Using the systematic corrections resulting from this final solution, we find the following reductions from *relative* to *absolute* parallax.

McCormick	$+0''.0053 \pm 0''.0021$	Yerkes	$+0''.0084 \pm 0''.0022$
Allegheny	$+0''.0076 \pm 0''.0017$	Mount Wilson	$-0''.0010 \pm 0''.0014$

The systematic corrections to the absolute magnitudes are in general very small except in the case of the giant G and M stars and the very brightest F stars, most of which are Cepheids or pseudo-Cepheids. For these stars, however, the probable errors are rather large on account of the smallness of the parallaxes. In the previous derivation of the reduction tables for the determination of absolute magnitudes it was found that in the case of the giant G stars the results from trigonometric parallaxes and parallactic motion differed considerably. The determination of the mean

absolute magnitudes based on parallactic motion would make these stars considerably brighter than the results from the trigonometric parallaxes indicate. A weighted mean was accordingly used. The discordance is now nearly eliminated by the application of the systematic corrections to the trigonometric parallaxes. The same holds for the brightest F stars, for which originally the measured parallaxes were not used at all, the result from parallactic motion having much higher weight. For the giant M stars the original determination was based mainly on the peculiar motion, and the present results seem to indicate that for these stars as a whole the system is nearly correct.

As the final result of this investigation, we may say that the system of spectroscopic parallaxes in *Mt. Wilson Contribution*, No. 199, as a whole, is correct within the errors of the quantities involved, and that there is no need at present to apply corrections to the system. The systematic corrections to the trigonometric parallaxes determined at the McCormick, Allegheny, Yerkes, and Mount Wilson observatories have been found to agree satisfactorily with those of van Maanen and Miss Wolfe, and, as the method used here gives absolute corrections, it can be regarded as probable that after these corrections have been applied there remain no appreciable outstanding constant errors in the resulting absolute parallaxes.

Mount Wilson Observatory
September 1921

NEW MEASUREMENTS OF STELLAR RADIATION

By W. W. Coblentz

ABSTRACT

Relative total radiation of 27 bright stars, down to magnitude 3.8.—New measurements which were recently made with the 40-inch reflector of the Lowell Observatory at Flagstaff, using a vacuum thermo-couple, are given in a table. These confirm the result obtained at Mount Hamilton in 1914 that the total radiation from red stars is 2.5–3 times as much as that from blue stars of the same visual magnitude.

Distribution of energy in the spectra of some bright stars was determined by the use of a series of filters which isolated various spectral regions. In this *preliminary note* merely the general result is reported that the *maximum emission* lies in the infra-red (0.7–0.9 μ) for stars of types K and M, and in the ultra-violet for stars of types B and A. The corresponding *black body temperatures* vary from 3000° C., for the red M stars to 9000° or 10,000° C., for the blue B stars. The *percentage transmission through 1 cm of water* is given for 22 stars and varies from 81 for B stars to 40 for M stars.

In a previous paper[1] data were given on a comparison of stellar radiometers and radiometric measurements of stars as observed at an altitude of about 4000 feet at Mount Hamilton, California, with the Crossley 36-inch reflector of the Lick Observatory. Quantitative measurements were made of stars down to magnitude 5.3 and qualitative measurements to magnitude 6.7. It was found that red stars emit from 2.5 to 3 times as much *total* radiation as blue stars of the same visual magnitude.

These observations were verified by an independent method which consisted in measuring the transmission of stellar radiation through a 1-cm cell of water, having quartz windows. By this means it was shown that, of the *total* radiation emitted, blue stars have about two times as much *visible* radiation as yellow stars, and about three times as much *visible* radiation as red stars.

At various times during the past seven years attention was given to the improvement of the stellar radiometers,[2] galvanometers, etc. The various subsidiary data will be published in a complete paper dealing with the whole subject.

The object of this paper is to give a preliminary survey of the results of new stellar radiometric measurements, made at a much

[1] Coblentz, *Bulletin of the Bureau of Standards*, **11**, 613, 1914.

[2] *Bulletin of the Bureau of Standards*, **13**, 423, 1916; **14**, 532, 1918; **16**, 253, 1920; *Journal of the Washington Academy of Sciences*, **6**, 473, 1916.

higher altitude, 7300 feet, at Flagstaff, Arizona, with the 40-inch reflector of the Lowell Observatory. This was made possible through the generosity of Drs. V. M. Slipher and C. O. Lampland. Not only was an otherwise busy program interrupted, but furthermore, Dr. Lampland personally operated the telescope, thus insuring speed and efficiency in accomplishing results. It is a pleasure to record here my grateful acknowledgments for the many courtesies accorded by various members of the staff of the Lowell Observatory.

The object of the present investigation was (1) to verify previous results; (2) to measure the intensities of radiation of bright stars in the region of 0 hours to 12 hours in right ascension, not previously measured; and (3) to determine the feasibility of the method of obtaining the distribution of spectral energy of stars by means of transmission screens which, either singly or in combination, are placed in front of the vacuum thermo-couple. By means of these thermo-couples, measurements were made on the intensities of radiation of thirteen bright stars not observed in 1914, thus completing the survey of the whole sky. A total of thirty celestial objects was measured, including Venus and Mars.

By means of a series of transmission screens (of yellow and red glass, of water, and of a thick plate of quartz) wide spectral regions were isolated and the intensities of radiation in the spectrum from 0.3 μ to 0.43 μ; 0.43 μ to 0.6 μ; 0.6 μ to 1.4 μ; 1.4 μ to 4 μ; and 4 μ to 10 μ were determined. In this manner the distribution of energy in the spectra of sixteen stars was determined, and thus was obtained for the first time an insight into the intensities of radiation in the complete spectrum of a star.

By means of this device it was found that in the stars of types B and A the maximum intensity of radiation lies in the ultra-violet (0.3 μ to 0.4 μ) while in the cooler stars of types K and M the maximum emission lies at 0.7 μ to 0. 9 μ in the infra-red. From this it appears that the black-body temperature (i.e., the temperature which a black body would have to attain in order to emit a similar distribution of relative spectral energy) varies from 3000°C. for red M stars to 9000° or 10,000°C. for blue B stars.

The observing station being much higher than previously used, the atmospheric scattering of light was greatly reduced and consequently the transmissions in the violet are somewhat higher than previously observed when the water-cell was interposed.

TABLE I

TRANSMISSION OF STELLAR RADIATION THROUGH A 1-CM LAYER OF WATER

Star	Magnitude Harvard Revision	Deflection for a Galvanometer Sensitivity of $i=1\times10^{-10}$ Amperes	Spectral Type	Percentage of Transmission	Remarks
θ Orionis (Nebula)	....	±0.02 cm	......	......	
ε Orionis........	1.75	0.66	B	81	
γ Orionis........	1.70	0.64	B2	82	
α Leonis.........	1.30	0.77	B8	......	
β Tauri..........	1.78	0.78	B8	79	
β Canis Minoris..	3.09	0.38	B8	......	
β Orionis........	0.34	2.89	B8p	63*	Rigel, spectroscopic binary
β Lyrae.........	Var.	0.26	B8p	......	
α Geminorum....	{1.99, 2.85}	0.78	A	82	Castor
α Lyrae.........	0.14	3.60	A	75	Vega
γ Geminorum....	1.93	0.71	A	......	
α Canis Majoris..	{−1.58, 8.5}	10.62	A	65*	Sirius, binary
α Cygni.........	1.4	1.34	A2	76	Deneb
α Aquilae........	0.9	1.88	A5	71	Altair
α Persei.........	1.90	0.75	F5	74	
α Canis Minoris..	{0.48, 13.5}	2.67	F5	64*	Procyon, binary
α Aurigae........	0.21	4.91	G	57	Capella
ε Geminorum....	3.18	0.58	G5	66	
β Geminorum....	1.21	2.19	K	58	Pollux
α Boötis.........	0.24	8.10	K	47	Arcturus
γ Leonis.........	{2.61, 3.80}	0.99	K	......	
α Tauri..........	1.06	6.03	K5	42	Aldebaran
λ Aquarii........	3.84	0.75	Ma	47	
μ Geminorum....	3.19	2.11	Ma	41	
β Andromedae....	2.37	2.45	Ma	41	
α Orionis........	0.92	15.00	Ma	34	Betelgeuse
α Scorpii........	1.22	8.82	Map	33*	Antares, spectroscopic binary
β Pegasi.........	2.61	2.96 cm	Mb	38	

However, all the data verify previous measurements showing that blue stars emit less infra-red radiation than do red stars of the same visual magnitude. Moreover, observations made on the same night (same weather conditions) are consistent in showing

small gradations in the infra-red component of the radiation, corresponding with the small gradations (say B2 and B8) in spectral types.

For binary stars having companions of low luminosity, transmissions of the water-cell are low, indicating that the companion stars emit considerable infra-red radiation. See Table I.

It was found that even in red stars the component of spectral radiation of wave-lengths greater than 4 μ is only from 1 to perhaps 10 per cent of the total. From this it would appear that in future work it may be permissible to use vacuum thermo-couples with thin quartz windows instead of fluorite, thus saving expense and possible leakage by constructing the container of quartz.

WASHINGTON, D.C.
October 24, 1921

SOLAR CONSTANT, SUN-SPOTS, AND SOLAR ACTIVITY

By ANDERS ÅNGSTRÖM

ABSTRACT

Variation of solar constant with number of sun-spots.—A study of 205 observations for 1915–1917 furnished by C. G. Abbot suggests that the solar constant S does not increase regularly with any power of the sun-spot number N, but rather reaches a maximum value for N between 100 and 160, approximated in accordance with the formula $S=1.903+0.011\sqrt{N}-0.0006N$. The number of observations for high value of N is, however, too small to be decisive.

A comparison of the values of the solar constant computed by Abbot and his collaborators with the sun-spot numbers given by Wolfer shows, if mean values for the separate years are considered, a close connection between the two phenomena. Thus a high sun-spot number seems to correspond to a high value of the solar constant and vice versa. The relation is apparently not a linear one; and inquiring into the exponent which applied to the sun-spot numbers gives the highest correlation between the two phenomena, I have found this exponent to be very nearly equal to $\frac{1}{2}$.[1]

If we consider only yearly mean values, the solar constant S seems then to be given with good approximation by the relation:

$$S=1.903+0.0055\sqrt{N}\,, \tag{1}$$

where N is the number of sun-spots according to Wolf-Wolfer. The mean difference between observed and computed values during the epoch 1905–1917 is less than 0.01, i.e., less than 0.5 per cent of the value of the solar constant itself, as may be seen from Table I.

As the question regarding the probable connection between solar constants and sun-spot numbers is an important one for the solution of many problems connected with the climate of past epochs and with the constitution of the sun, it seems worth while to enter a little more closely into the question.

[1] *Geografiska Annaler*, H. 1, 1920.

For the low values of N which are obtained by taking averages during a considerable time the relation (1) seems to hold with good approximation. But is this true also for a wider range of values of N? If we consider the values of S and N corresponding to individual days, what will be the relation between them?

In order to answer these questions, I have divided the values of the solar constant during the years 1915–1917, kindly put at my disposal by Dr. Abbot, in groups according to the corresponding sun-spot numbers, as is seen from Table II. The years named are selected from the point of view that the sun-spot numbers have varied during that time over a considerable range ($0 < N < 256$), and also because the variation in the yearly mean values is small, and consequently should not have introduced any complications. The values of the solar constant corresponding to the various groups of sun-spot frequencies are given in Table II together with the numbers of values of which they represent the means. On the basis of this table, Figure 1 has been drawn, where the values of the solar constant are plotted against sun-spot numbers.

The table and figure seem now to suggest the following conclusions. With increasing sun-spot numbers, the solar constant first increases in order to reach a maximum, which seems to correspond to a sun-spot number of between 100 and 160 (maximum difficult to locate exactly); thereafter, the solar constant decreases with an increase in sun-spot numbers. The relation between solar constant and sun-spot numbers is apparently governed by two separate phenomena. The one causes an increase of the solar constant with increasing sun-spot numbers, and this effect predominates for the small values of N; the other effect goes in the opposite direction, and predominates for large values of N. Let us start from the assumption that the relation between solar constant and sun-spots for small values of N may be given by the expression

$$S = 1.903 + k\sqrt{N}, \tag{2}$$

obtained in its general features by the study of the yearly mean values. We may assume the other effect, which evidently must

depend upon a function of higher order of N than $N^{\frac{1}{2}}$, to be of the form N^{a}, where a is to be determined from the observations. The expression for the relation then takes the form:

$$S = 1.903 + k\sqrt{N} - cN^{a}. \tag{3}$$

A comparison with the tabulated values gives us the following approximate values for the constants:

$$k=0.011,\ c=0.0006,\ a=1 \text{ (approx.)}.$$

The smooth curve of Figure 1 is computed from (3) on the basis of these values of a, k, and c.

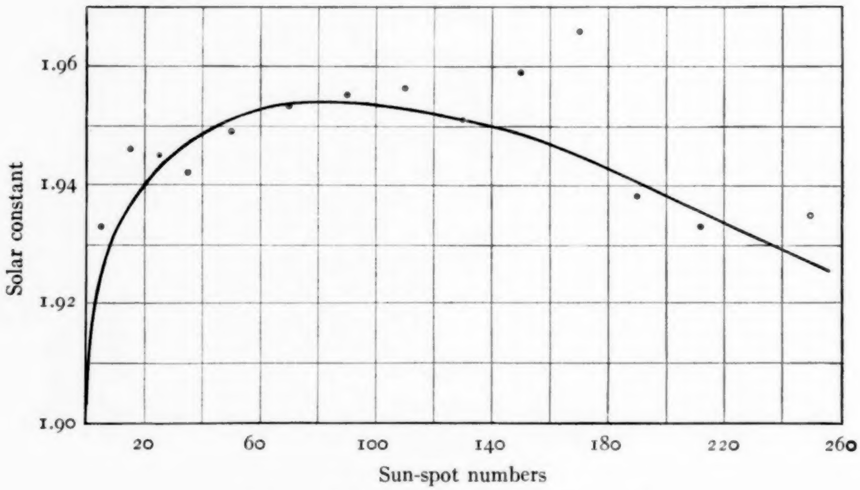

FIG. 1.—Solar constant and sun-spot numbers

It is clear that too much weight ought not to be attached to a result based upon a number of observations which, especially for large values of N, is very limited. In fact, the decrease of the solar constant with increasing N after a maximum value is reached is based upon ten observations, of which, according to Abbot, four are excellent (*e*), two very good (*vg*), three good (*g*), and one poor (*p*). It is to be noted, however, that the value marked (*p*) is the highest one of them all, namely 1.971, and consequently it has raised and not lowered the mean value. But, on the other

hand, the result seems to me to be in remarkable agreement with consequences to be expected from the nature and constitution of sun-spots. The sun-spots themselves are dark relative to the surrounding parts of the sun's disk. The decrease in brightness varies between wide limits, but according to E. Liais[1] and M. Gouy and L. Thollon,[2] the larger sun-spots generally have a brightness less than 20 per cent of the normal. The percentage of heat radiation is somewhat larger. S. P. Langley[3] gives it as about one-half of the normal for the photosphere. It is evident that the spot-covered area itself cannot contribute to the increase in the solar constant. The observations of individual sun-spots show, however, that the dark area of the spot generally is surrounded by an outer region, where the brightness for a number of special lines is more intense than the normal for the surface. At this area the protuberances and flocculi are especially strongly developed, surrounding the inner and darker part by a kind of luminous corona of more or less symmetric form. If we assume the point of view expressed by Abbot, Fowle, and Aldrich, namely, that the increase in the solar constant is caused mainly by an increase in the vertical convection through which hot material is carried to the surface of the sun, we must conclude that the seat of this "intensified" radiation must be located especially at peripheric rings round the sun-spots. It seems very natural, then, that the areas of these rings must increase less rapidly than the area of the sun-spots, viz., the Wolf-Wolfer sun-spot numbers. On the other hand, it seems as natural that the simultaneous decrease of the solar constant is proportional to the spot area, which as we know represents dark regions of the sun's surface for luminous as well as dark heat radiation. The measured area of the sun-spots is, it is true, scarcely more than a few thousandths of the sun's disk. But to this area we must probably add a great part of smaller spots of granular structure which never are counted as real sun-spots, but the area of which increases and decreases with the sun-spot number itself.

The data available at present are too few to give a very definite idea of the form of the relation between solar constant and

[1] *Mémoires de la Société des Sciences de Cherbourg*, **12**, 1866.

[2] *Comptes Rendus*, **95**, 1834–1836, 1882.

[3] *Monthly Notices*, **37**, 5, 1876.

sun-spot number. It is with all probability a function dependent not only on the area of sun-spots, but also on their number, intensity, and constitution in general. *So much seems clear, however, from the previous discussion, that this function is not a linear one; the opinion that an increase of sun-spots is also accompanied by an increase in the solar constant must be subjected to a revision. In all probability the solar constant passes through a maximum with increasing sun-spot area*, after which a continuous decrease of the solar constant takes place when the area increases.

From our material at present it therefore does not seem permissible to assume that very high values of the solar constant in the past have accompanied a very high value of the frequency of sun-spots, but, on the contrary, that the solar constant at certain epochs possibly may have had a very low value on account of a great frequency of sun-spots. If we dared to extrapolate the function given above to represent the conditions at very high sun-spot frequencies, we should find that a sun-spot number of 1,000 corresponds to as low a solar constant as about 1.66, or a decrease from the normal by about 15 per cent. As such high sun-spot frequencies have not yet occurred, so far as we have observed, we then enter into the region of hypotheses.

Further determinations of the solar constant, especially at the times of maxima of sun-spots, will be of great interest in throwing more light upon the questions raised above. The foregoing discussion seems to the author to form an additional support for the opinion that the variations of the solar constant, observed by Abbot and his collaborators, are real and the sun consequently is a variable star.

Recently doubts as to the reality of the variations of the solar constant have been raised by G. Granqvist on account of a correlation found to exist between the values of the solar constant and the values of the atmospheric transmission[1]. In a recently published paper[2] I consider that I have shown: (1) that in the case of Abbot's values the correlation pointed out by Mr. Granqvist

[1] *Kosmos*, Stockholm, 1921.

[2] "Är solens strålning variabel?" *Tidskrift för elementär matematik fysik och kemi.* Alb. Bonnier, Stockholm, 1921.

practically disappears, if one accounts for the probable variation of the solar constant itself during the time of observations; and (2) that the connection between solar constant and transmission in the case of the observations of Mr. Granqvist probably is caused by variations in the atmospheric transmission during the time of observations, i.e., too high a value of the solar constant is obtained when, in view of the increasing transmission, too low a value is computed for the transmission, and vice versa.

TABLE I

Years	Solar Constant (S)		Sun-Spot Numbers (N)	O.–C.
	Observed	Computed	(Wolfer)	
1905	1.956	1.946	63	+0.010
1906	1.942	1.945	58	−0.003
1908	1.936	1.944	55	−0.008
1909	1.918	1.940	46	−0.022
1910	1.921	1.928	21	−0.007
1911	1.921	1.922	3	−0.001
1913	1.904	1.909	1	−0.005
1914	1.956	1.919	9	+0.037
1915	1.952	1.946	62	+0.006
1916	1.946	1.942	50	+0.004
1917	1.960	1.961	113	−0.001
Mean difference				0.009

TABLE II

Sun-Spot Number (N)	Mean Solar Constant (S)	Number of Observations (n)
0–9	1.933	5
10–19	1.946	12
20–29	1.945	17
30–39	1.942	17
40–59	1.949	35
60–79	1.953	33
80–99	1.955	27
100–119	1.956	23
120–139	1.951	11
140–159	1.959	10
160–179	1.966	5
180–199	1.938	2
200–224	1.933	3
225–275	1.935	5

STOCKHOLM
May 1921

THE ORBITS OF THE SPECTROSCOPIC BINARIES 1 HYDRAE AND 75 CANCRI[1]

By R. F. SANFORD

ABSTRACT

Orbits of the spectroscopic binaries, Boss 2227 = 1 Hydrae and Boss 2447 = 75 Cancri.—These stars are of classes F1 and G2 and of visual magnitudes 5.7 and 6.0, respectively. Twenty-seven spectrograms of each give for the periods, 1.563 and 19.46 days; for e, 0.05 and 0.206; for K, 30.3 and 20.2 km/sec.; and for γ, +71.3 and +12.3 km/sec., respectively. Attention is called to the large γ for Boss 2227, and to the fact that Boss 2447 is a dwarf star. Radial velocity-curves are shown.

A previous paper[2] discusses the orbits of seven spectroscopic binaries; the present note gives the orbits of two additional binaries. The general remarks which precede the discussions of individual stars in the first paper are equally applicable here and need not be repeated.

The data in Table I, except for the last column which gives the number of revolutions of the star in its orbit between the first and last observations, are taken from the list of spectroscopic parallaxes of 1646 stars.[3] Figures 1 and 2 show the radial velocity-curves. In the following pages the derivation of the orbits of these two spectroscopic binaries is discussed.

TABLE I

Name	Vis. Mag.	α (1900)	δ (1900)	Spectral Class	Vis. Abs. Mag.	μ	$\pi_{sp.}$	No. Rev.
Boss 2227–1 Hydrae	5.7	$8^h 19^m.6$	− 3°26′	F1	+3.6	0″.216	0″.038	340
Boss 2447–75 Cancri	6.0	9 2.9	+27 3	G2	+4.1	0.404	0.042	59

BOSS 2227

Measures of the first two spectrograms of this star by Professor H. C. Wilson showed that its radial velocity is variable. Accordingly observations were started immediately for the determination

[1] *Contributions from the Mount Wilson Observatory*, No. 221.

[2] *Mt. Wilson Contr.*, No. 201; *Astrophysical Journal*, **53**, 201, 1921.

[3] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 13, 1921.

of the elements of its orbit. Twenty-seven suitable spectrograms are listed in Table II. The derivation of the correct period presented some difficulty at first because of its shortness. Finally,

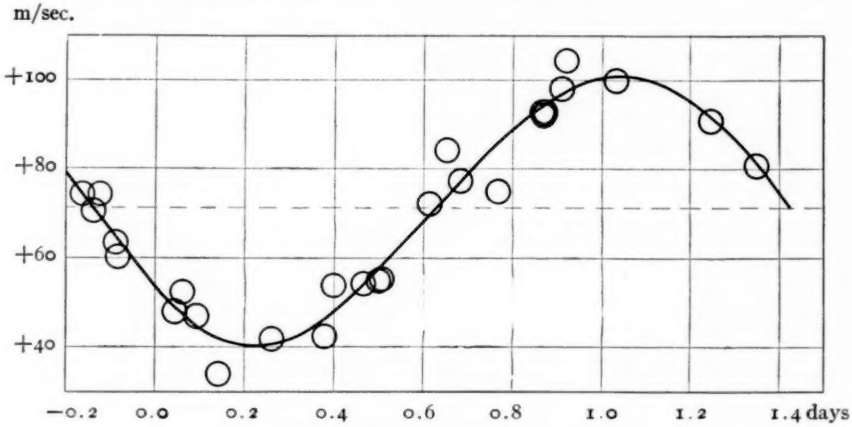

FIG. 1.—Radial velocity-curve for Boss 2227

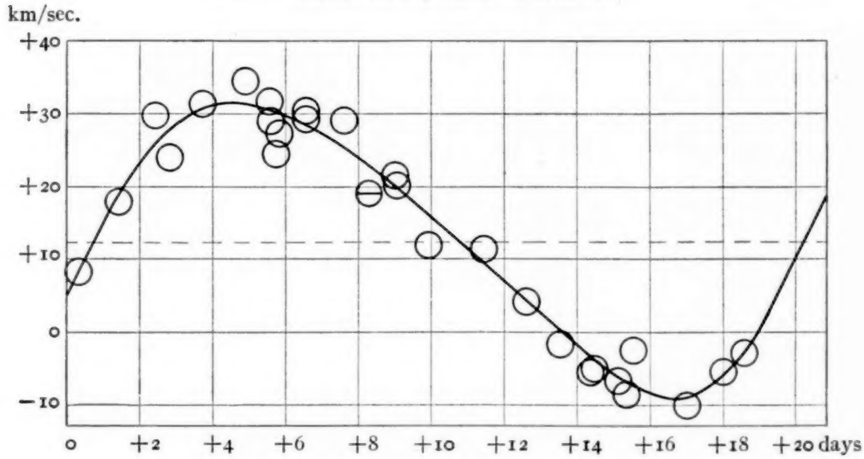

FIG. 2.—Radial velocity-curve for Boss 2447

all observations were gathered into one period in a satisfactory manner when $P=1.562975$ days was used. Since approximately 340 revolutions of the star in its orbit occurred between the first and last observations it seemed needless to endeavor to correct the period, and therefore the value given above has been taken as final.

With preliminary elements derived by Russell's method, an ephemeris was computed and the residuals (O–C) derived. The spectrum is of Class F1, with only one set of lines which at best yields a velocity of only fair precision and which varies so in quality

TABLE II

OBSERVATIONS OF BOSS 2227

Plate No.	Date		G.M.T.	Phase	Weight	Velocity	O−C
						km/sec.	km/sec.
γ 8860......	1919	Nov. 12	0^h43^m	$0\overset{d}{.}062$	0.75	+ 52.2	+ 4.9
9109......	1920	Apr. 4	15 11	0.870	0.75	+ 92.6	− 1.8
9666......		Oct. 26	0 19	0.501	1.00	+ 54.6	− 2.7
9687......		29	0 24	0.380	0.75	+ 42.4	− 3.9
C 741......		Nov. 1	0 40	0.262	0.75	+ 41.8	+ 1.3
γ 9705......		2	0 13	1.246	1.00	+ 90.6	− 1.2
9723......		20	0 36	0.506	0.50	+ 55.0	− 2.8
9732......		20	23 58	1.479	0.75	+ 60.3	− 3.6
9738......		21	22 47	0.867	1.00	+ 92.8	− 1.4
9747......		23	1 06	0.401	0.50	+ 53.7	+ 5.8
C 778......		24	1 28	1.416	1.00	+ 70.9	− 0.8
γ 9809......		Dec. 22	22 55	0.614	1.00	+ 72.1	+ 2.4
9893......	1921	Feb. 15	16 47	0.653	0.50	+ 84.0	+ 9.9
9897......		17	15 22	1.031	0.75	+ 99.9	− 0.7
9934......		21	18 20	0.466	0.75	+ 54.1	+ 0.4
C 897......		22	18 29	1.473	1.00	+ 63.5	− 1.2
γ 9940......		23	18 32	0.910	1.00	+ 98.0	+ 1.2
9957......		25	19 46	1.400	0.75	+ 74.2	− 0.2
9995......		Mar. 16	14 50	1.439	1.00	+ 74.4	+ 5.2
9999......		16	18 52	0.045	1.00	+ 48.0	− 0.9
10003......		17	14 42	0.871	1.00	+ 92.8	− 1.6
10012......		19	15 42	1.349	0.75	+ 80.6	− 0.2
10046......		24	15 19	0.095	1.00	+ 46.9	+ 1.8
10091......		Apr. 17	16 25	0.682	0.50	+ 77.3	+ 0.1
10099......		18	17 03	0.143	0.50	+ 33.9	− 8.0
10124......		25	17 40	0.920	1.00	+104.4	+ 7.0
10139......		28	17 04	0.768	0.75	+ 75.0	−11.0

from plate to plate that it has seemed advisable to assign weights to the measures of different plates. The best have been given weight unity and the rest weight 0.75 to 0.50, according to the inherent quality of the lines, or their quality as affected by photographic density, focus, etc., or a combination of these circumstances. The twenty-seven observations were converted into seventeen normal places, assigned proper weights and then used to correct the preliminary elements by the method of least squares. With the elements thus corrected the quantity Σpv^2 is about 75 per cent

of its value when derived with the preliminary elements. The residuals obtained by substitution of the unknowns in the equations of condition agree satisfactorily with the residuals from an ephemeris based on the final elements. In Table III are to be found preliminary elements, corrections, final elements, and their probable errors. The probable error of a single observation of velocity of weight unity is ±2.64 km/sec. The absolute magnitude (+3.6) is very nearly that of Class F stars of greatest frequency and is therefore typical.

TABLE III

Preliminary Elements		Corrections	Final Elements
P		....	1.562975 days
e	0.08	−0.029	0.051 ± .035
ω	119°	+4°.92	123.92 ± 34°.4
K	30.5 km/sec.	−0.22 km/sec.	30.28 ± 1.38 km/sec.
T	J.D. 2422650.073	+0ᵈ009	2422650.082 ± 0ᵈ132
γ	+71.7 km/sec.	−0.40 km/sec.	+71.3 km/sec.
$a \sin i$		....	650000 km
$\frac{m^3 \sin^3 i}{(m+m_1)^2}$		....	0.0045 ☉

BOSS 2447

This was announced as a spectroscopic binary in a list published by Adams and Joy.[1] The single set of lines appearing on its spectrograms, which show that its class is G2 and its absolute magnitude +4.1, is quite satisfactory for measurement on properly exposed plates. Table IV gives the data for the twenty-eight plates which have been obtained. With $P = 19.4589$ days all observations arranged themselves satisfactorily within a single period and furnished the basis for the preliminary elements. Since the observations extend over an interval of approximately sixty revolutions of the star in its orbit, this period was taken as definitive. By the method of least squares the other five elements were corrected, equal weights being given to all of the twenty-seven plates which were used. Plate γ 6662, whose residual stood out as the only large one, was arbitrarily rejected from the least-squares solution, and Plate γ 9997 was obtained after the elements were

[1] *Publications of the Astronomical Society of the Pacific*, **31**, 41, 1919.

derived. Data for both plates are given in Table IV, however. The quantity Σpv^2 derived from the elements, corrected as indicated by the solution given above, is 18 per cent less than for the preliminary elements, and substitution of the unknowns into the equations of condition gave residuals comparable with those

TABLE IV

OBSERVATIONS OF BOSS 2447

Plate No.	Date			G.M.T.	Phase	Velocity	O−C
						km/sec.	km/sec.
γ 6396	1917	Nov.	27	0^h01^m	9^d021	+21.6	+ 1.5
6591	1918	Jan.	22	22 51	7.598	+29.1	+ 3.8
6662*			30	22 19	15.574	+ 4.3	+11.8
6706		Feb.	28	20 43	5.589	+29.0	− 1.4
6894		May	1	16 59	9.057	+20.2	+ 0.3
6943			30	16 43	18.589	− 2.8	− 0.3
C 346	1920	Apr.	3	15 02	11.456	+11.4	+ 1.8
352			4	18 43	12.609	+ 4.2	− 0.1
356			5	17 00	13.537	− 1.6	− 1.8
γ 9123			6	15 22	14.469	− 5.0	− 1.2
9130			7	17 04	15.540	− 2.5	+ 4.9
9179		May	6	15 39	5.563	+31.7	+ 1.2
9186			7	15 35	6.560	+30.4	+ 2.0
C 410			7	15 35	6.560	+29.3	+ 0.9
γ 9255		June	4	16 16	15.130	− 6.7	− 0.5
9266†			6	15 58	17.017	−10.1	− 1.2
9272			7	15 56	18.016	− 5.4	+ 0.6
9667		Oct.	26	0 52	2.813	+24.0	− 3.9
9688			29	1 03	5.821	+27.3	− 2.7
9767		Nov.	26	1 05	14.367	− 5.5	− 2.1
9774			27	0 53	15.359	− 8.6	− 1.8
9876	1921	Jan.	28	20 14	0.326	+ 8.2	− 0.1
9886			29	22 21	1.414	+17.9	− 0.8
9889			30	22 33	2.422	+29.8	+ 3.9
9927		Feb.	20	16 36	3.716	+31.4	+ 0.8
9935			21	20 56	4.896	+34.5	+ 3.3
C 896			22	17 10	5.739	+24.4	− 5.7
γ 9969			26	21 46	9.930	+11.9	− 4.4
9997‡		Mar.	16	17 06	8.277	+19.2	− 3.7

* Not used in least-squares solution.

† Poor plate.

‡ This plate obtained after elements were derived.

derived from an ephemeris calculated with the corrected elements, which are therefore adopted as final. The preliminary elements, corrections, final elements, and probable errors are given in Table V. The probable error of a single observation of velocity of weight unity is ±1.74 km/sec. The barred circle in Figure 2 represents the velocity from Plate γ 9997.

TABLE V

	Preliminary Elements	Corrections	Final Elements
P		....	19.4589 days
e	.200	+ .006	0.206 ± .002
ω	248°.5	+4°.00	252°.5 ± 7°.70
K	20.5 km/sec.	−0.29 km/sec.	20.21 ± 0.49 km/sec.
T	J.D. 2422426.555	+0ᵈ.079	2422426.634 ± 0ᵈ.384
γ	+12.49 km/sec.	−0.16 km/sec.	+12.3 km/sec.
$a \sin i$		....	5295900 km
$\frac{m^3 \sin^3 i}{(m+m_1)^2}$		....	0.0157 ⊙

MOUNT WILSON OBSERVATORY
October 1921

AN INVESTIGATION OF THE CONSTANCY IN WAVE-LENGTH OF THE ATMOSPHERIC AND SOLAR LINES[1]

By CHARLES E. ST. JOHN and HAROLD D. BABCOCK

ABSTRACT

Constancy in wave-length of atmospheric lines in the solar spectrum.—In 1915 Perot reported having found the wave-length of an O line considerably longer at noon than at sunrise and sunset. Since these lines are constantly used as standards of wave-length in solar observations, a study of the *wave-length as a function of the altitude of the sun* was made for a number of atmospheric lines in the B group at λ 6867, in the a group at λ 6276, and in the water-vapor band near λ 5900. Measurements of 25 plates show no indication of a variation greater than the accidental error. Moreover, spectrograms and data accumulated at Mount Wilson since 1911 give wave-lengths, some determined with reference to solar lines and some with reference to arc lines, which agree very closely, the difference from the general mean shown by any plate seldom exceeding 0.001 A. This negative result indicates the absence of high velocity *radial currents in the earth's atmosphere* and justifies the use of atmospheric lines as a reliable standard of reference even in work requiring the highest precision.

Constancy in wave-length of lines from the center of the solar disk.—While Evershed reported in 1919 remarkable variations amounting to several thousandths of an angstrom, the experience at Mount Wilson is that the more carefully the solar wave-lengths are compared with standard arc lines, the smaller the deviations from spectrogram to spectrogram become. Evidence from 13 plates is presented. These results prove that the *radial convection currents in the sun*, while not absent, are remarkably constant, apparently downward at high levels and upward, but small, at low levels.

In spectrographic observations on the solar spectrum for determining wave-lengths and displacements of the Fraunhofer lines with high precision, a desideratum is some means of procuring a simultaneous comparison spectrum under the same conditions of illumination as those obtaining for the spectrum under investigation. Provided their wave-lengths are constant, ideal conditions occur when atmospheric lines are used for reference in solar observations, since in the instrument the path of the light is identical for both classes of lines.

It has been generally assumed that under all practical conditions of solar observation the velocities of terrestrial atmospheric movements are of such an order that no measurable Doppler effect is produced. It would require motion in the line of sight of approximately 125 miles an hour to cause a displacement of 0.001 A

[1] *Contributions from the Mount Wilson Observatory*, No. 223.

at λ 6000. Some observations reported by Perot,[1] however, give very remarkable and surprising results, and if confirmed would show that the atmospheric lines are not constant in wave-length and hence not reliable as standards of reference. He measured with an interferometer the wave-length of an oxygen line in the B group at different hours of the day and concluded that the wave-length increased from morning to noon and decreased from noon to evening. He interprets his results as showing a recession of the absorbing centers from the surface of the earth with a radial velocity of about 3 km per second.

In view of the importance of the atmospheric lines in solar observations we have examined for change in wave-length during the day the atmospheric lines in three spectral regions, namely, in the B group at λ 6867, in the α group at λ 6276, and in the water-vapor band near λ 5900, and have supplemented these observations from spectrograms taken for other purposes through a series of years. For much of the measurement and reduction we are under obligation to Miss Ware, Miss Miller, and Miss Keener.

Oxygen lines in the B group.—On two occasions grating spectrograms of the center of the sun were taken in the first order of the 75-foot spectrograph of the 150-foot tower at intervals from sunrise to sunset. The wave-lengths of eight oxygen lines were obtained from solar lines whose wave-lengths were corrected for the earth's motions. The statistical results are given in Table I. The lines are identified by their Rowland wave-lengths in the first and fourth columns and in the other columns are given the deviations from the mean for the hours of observation. In Figure 1*A* the average deviations from the general mean are plotted against the corresponding altitudes of the sun.

From our investigation on solar wave-lengths in the international system we extract the data given in Table II. These measurements were made during June, 1919, with the interferometer attached to the Snow telescope. Our unpublished international values of solar lines were used as standards. The first column contains the preliminary wave-lengths in the international system of 28 oxygen lines in the B group. In the other columns are shown

[1] *Comptes Rendus*, **160**, 549, 1915.

TABLE I

RELATIVE WAVE-LENGTHS OF OXYGEN LINES OF THE B GROUP AT DIFFERENT HOURS OF THE DAY. GRATING SPECTROGRAMS

(Unit for residuals=0.001 A)

λ Rowland	February 25–26, 1919, P.S.T.			λ Rowland	June 5, 1919, P.S.T.						
	7:00	12:00	4:37		6:27	9:12	11:56	2:57	3:02	5:41	5:50
6879.288	+2	−3	0	6877.882	+4	0	−2	−2	0	0	0
6880.172	+6	+2	−7	6880.172	+4	+2	−2	−1	+1	−2	−2
6884.076	0	+2	−3	6884.076	+4	−4	−5	0	+1	+1	0
6886.000	+1	+2	−3	6900.199	+2	−4	−3	+2	+2	−2	+4
6913.448	−7	+5	+1	6905.271	+2	−2	+1	−1	0	−3	0
6914.337	−5	0	+6	6909.676	+1	0	0	−3	+1	−1	−1
6918.370	....	+1	−1	6913.448	+2	0	−1	−3	−1	0	−4
6919.250	....	−1	+2	6918.370	+1	0	+3	0	+2	−2	−1
Mean	−0.5	+1.0	−0.6	Mean	+2.5	−1.0	−1.1	−1.0	+0.6	−1.1	−0.5
Altitude	7°00′	46°40′	10°20′	Altitude	17°40′	51°30′	78°10′	49°40′	48°40′	16°10′	14°20′

the deviations from the mean values for altitudes varying from 12°50′ to 75°40′. The observed mean wave-length at the highest altitude is 0.0012 A less than the general mean.

Oxygen lines of the α group.—On June 7 and 9, 1919, spectrograms of the center of the sun were taken with the 75-foot spectrograph at intervals from 6:00 A.M. to 6:00 P.M. Eight oxygen lines of the α group were measured in terms of solar standards. After correction of the reference lines for the earth's motions, the mean wave-length at noon exceeded the mean for the two lowest altitudes by 0.0016 A. The numerical data are given in Table III; the relative results are plotted in Figure 1*B* against the hours of observation.

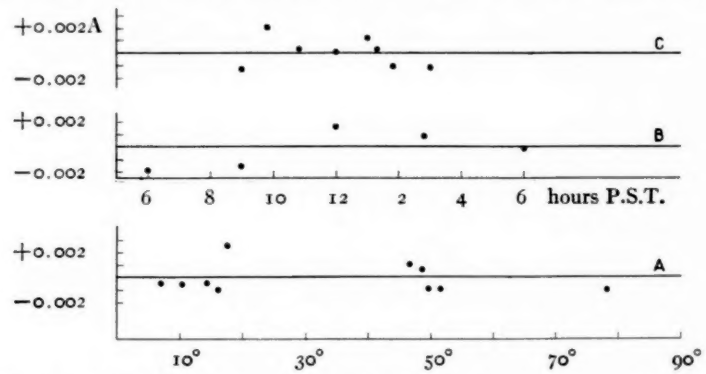

FIG. 1.—*A*. Deviations of oxygen lines in B group from mean wave-length; for different altitudes of sun.

B. Deviations of oxygen lines in α group from mean wave-length; at different hours of the day. Solar standards.

C. Same as *B*, referred to absorption lines of iodine as standards.

In order to eliminate solar standards and obtain an independent determination, spectrograms of sunlight filtered through iodine vapor were taken with an interferometer using two absorption lines of iodine as the fixed references. These observations were made in the Pasadena laboratory in April, 1919, with an etalon of 20 mm separation giving a mean order of interference of 63400 for the eight measured oxygen lines of the α group. The deviations from the mean wave-length at the hours of observation are plotted in Figure 1*C*. The wave-length at noon shows no deviation from the mean of the series.

TABLE II

RELATIVE WAVE-LENGTHS OF OXYGEN LINES OF THE B GROUP AT DIFFERENT HOURS OF THE DAY. INTERFEROMETER SPECTROGRAMS

(Unit for residuals = 0.001 A)

June 6, 15, 16, 1919, P.S.T.

λ I.A.	June 6 9:08	June 6 10:32	June 6 12:36	June 16 3:40	June 15 5:11	June 16 6:03
6872.265	−3	−1	+1	+2	+3	−3
6872.861	+1	−3	−3	+3	0	....
6873.816	−2	0	0	0	+4	−2
6874.671	−2	+1	−3	−2	+5	0
6875.608	0	0	0	0	+3	−5
6876.733	−2	+3	0	−2	−2	+1
6877.655	+2	+2	−2	−3	+1	−1
6879.058	+2	+3	−3	+2	−1	−10
6879.947	+4	0	−3	−1	−1	−5
6883.850	+1	+1	0	−1	+1	−6
6885.772	−1	+1	−1	−1	+4	−4
6886.761	−1	+1	−2	0	+3	−3
6888.967	+1	+1	−1	0	0	−3
6889.921	−2	0	0	+2	−3	....
6892.389	−6	+3	−3	+2	−1	....
6893.328	0	0	−5	−2	+6	−4
6896.056	0	+2	+2	−2	−3	−2
6896.984	+1	0	0	+1	−3	−3
6899.972	−1	0	−1	+2	−1	+2
6900.887	−2	+1	−4	+0	+4	−5
6904.137	−1	+2	−2	−2	0	....
6905.137	−1	+2	−2	−2	0	....
6908.553	+1	+1	−3	−1	+3	−2
6909.450	−1	+2	−2	−1	+1	−3
6913.219	0	+3	+1	−4	+1	....
6914.109	0	+5	0	−4	+1	....
6918.140	−2	+2	−2	+1	−3	....
6919.022	+1	0	−1	−1	+3	....
Mean	−0.4	+1.1	−1.4	−0.5	+1.0	−2.9
Altitude	50°40′	67°20′	75°40′	41°10′	22°30′	12°50′

From spectrograms and data accumulated since 1911, when the *a* band was first studied at Mount Wilson, a yearly record is available. The results for eight lines are given in Table IV. The mean wave-lengths for the eleven-year period are in the first column and those for the separate years in the succeeding columns. The mean

wave-length of the group is shown at the bottom of the table. The spectrograms were taken in various months, on various days, and at various hours. It would seem that frequently occurring disturbances should have been detected; but on none of the scores of spectrograms measured is there definite evidence of radial movements of the absorbing regions of the atmosphere. The difference from the general mean shown by any plate seldom exceeds 0.001 A.[1]

TABLE III

WAVE-LENGTHS OF OXYGEN LINES OF THE a GROUP AT DIFFERENT HOURS OF THE DAY. GRATING SPECTROGRAMS

June 7–9, 1919, P.S.T.

λ I.A.	June 7 6:00	June 7 9:00	June 7–9 12:00	June 9 2:50	June 9 6:00
6280.404.........	.403	.402	.406	.404	.403
6281.187.........	.185	.186	.189	.189	.186
6281.964.........	.962	.962	.966	.966	.963
6290.230.........	.229	.230	.232	.230	.229
6295.187.........	.185	.184	.188	.187	.188
6295.969.........	.967	.968	.970	.971	.968
6302.009.........	.006	.006	.010	.009	.010
6302.770.........	.768	.769	.772	770	.771
Mean .215........	.2131	.2134	.2166	.2158	.2148
No. of plates......	2	1	3	2	4
Altitude..........	12°30′	49°10′	78°40′	51°20′	12°40′

Water-vapor lines near λ *5900*.—To complete the investigation on atmospheric lines, grating spectrograms of the center of the sun's disk were taken at high and low sun in the region of the rain-band near D_1 and D_2. The results of the measures of eleven lines on twenty-four spectrograms are shown in Table V. Solar lines were used as standards. The mean wave-lengths in the international system are in the first column, weighted according to the number of exposures. For the eleven lines the wave-length at high sun exceeds the mean at low sun by 0.001 A.

[1] This confirms and extends in time other observations on the a group. Royds, *Annual Report, Kodaikanal Observatory*, 1917; St. John and Babcock, *Publications of the Astronomical Society of the Pacific*, **31**, 178, 1919.

TABLE IV

Yearly Record for Atmospheric Lines of the a Group

Mean Wave-Lengths I.A. Units	Standards of Reference										
	Solar Lines Corrected for the Earth's Motions								Iron Arc Lines Used as Standards		
	At Center			At Limb, Corrected for Limb-Center Shift							
	1911	1912	1913	1914	1915	1916	1917	1918	1919	1920	1921
6280.400	.400	.400	.403	.403	.400	.397	.400	.402	.402	.398	.403
6281.186	.183	.185	.185	.186	.184	.183	.185	.185	.186	.187	.188
6281.963	.961	.960	.963	.964	.966	.962	.963	.963	.963	.963	.964
6290.229	.230	.229	.228	.230	.229	.228	.226	.229	.229	.229	.229
6295.186	.187	.183	.184	.186	.185	.184	.187	.182	.187	.185	.185
6295.968	.970	.967	.968	.969	.968	.967	.967	.965	.968	.968	.970
6302.008	.009	.007	.010	.012	.006	.008	.005	.005	.008	.007	.009
6302.771	.771	.771	.769	.776	.772	.768	.771	.767	.770	.772	.772
Mean .214	.214	.213	.214	.216	.214	.212	.213	.212	.214	.214	.215
No. of plates	8	2	2	3	4	2	2	2	22	13	2

TABLE V

WAVE-LENGTHS OF WATER-VAPOR LINES AT DIFFERENT HOURS OF THE DAY

(March and June 1921)

λ I.A.	P.S.T. of Observation			
	7:03	11:40	3:16	5:40
5885.982	.980	.983	.984	.980
5887.227	.226	.227	.230	.226
5887.665	.664	.668	.663	.664
5892.402	.396	.405	.403	.402
5909.001	.998	.000	.004	.002
5913.999	.998	.996	.004	.999
5919.058	.056	.059	.059	.057
5919.646	.644	.647	.646	.646
5924.276	.275	.275	.278	.276
5932.097	.096	.097	.098	.097
5932.788	.788	.788	.788	.788
Mean .4675	.466	.468	.469	.467
Altitude	16°	61°	35°	13°

When the observations on atmospheric lines at high sun are compared with the mean for all altitudes, the residuals, high sun *minus* mean, are as follows:

Source	Lines Observed	Altitude High Sun	Residuals
Table I	B group	78°10′	−0.0010 A
Table II	B group	75 40	−0.0012
Table III	α group	78 30	+0.0016
Table IV	Water-vapor	61 20	+0.0005
Mean	....	....	0.0000

We are unable to account for the wide divergence between our measures and those of Perot. Within the error of measurement our series of observations show no variation in the wave-lengths of the terrestrial lines with the altitude of the sun. It will be recalled that Dunér,[1] in his determination of solar rotation in 1887–1889, used the terrestrial line λ6302.209 as a fixed point of reference, and in 1899–1901 the other component of the pair, λ6302.975. If the terrestrial atmosphere in the absorbing region is subject to radial velocities of 3 km per second his consistent

[1] *Über die Rotation der Sonne*, Upsala, 1906.

values are remarkable, particularly for the highest latitude where the observed rotational velocity was only 0.39 km per second. It must be admitted that radial velocities in the earth's atmosphere of the order of 3 km per second, if they do occur, are extremely rare and very local. It seems to us more probable that a different interpretation should be sought for Perot's observations. His interpretation requires for the absorbing centers a radial velocity of 6700 miles per hour. That such enormous radial velocities occur even in attenuated regions of the earth's atmosphere is open to question, and it is more doubtful still that the radial component would remain constant for a month over a given locality. From our observations we feel justified in using the atmospheric lines as reliable standards of reference, even in work requiring the highest precision.

On constancy of wave-length of solar lines.—Jewell[1] was the first to raise the question of variability in the wave-lengths of the solar lines. He says,

> Another effect of this investigation may be to make the lines of the solar spectrum step down from the commanding position which they have occupied, as standards of reference.

At the period of Jewell's investigation the instability of many lines in the arc spectra of metals had not been recognized. Moreover, the records of the spectrograms taken by Rowland and used by Jewell were not sufficiently detailed to determine corrections depending upon the earth's motions. Under these conditions the conclusion reached by Jewell loses much of its weight. Evershed[2] reports that

> a continuous series of sunlight and Fe spectra was taken to test the constancy of the sun-arc displacement. Confining attention to the region 4337–4531 and to lines not subject to pole-effect in the arc, it was found that some remarkable variations occurred amounting to several thousandths of an angstrom. The variations are of two kinds: a general change for all the lines in the region studied, and a change affecting particular lines or groups of lines.

He says,

> These displacements may be observed at the center of the disk, but up to the present they have not been found very near the limb. It appears therefore that, unlike the displacements in the penumbra of spots, they may be

[1] *Astrophysical Journal*, **3**, 113, 1896.

[2] *Annual Report, Director, Kodaikanal and Madras Observatories*, 1919, p. 2.

due to movements normal to the surface or having a component normal to the surface.

At Mount Wilson it has been found that in the determination of solar wave-lengths from arc standards, the more carefully the iron arc is controlled, the more completely instrumental displacements are eliminated, the more accurately the image of the sun is centered,[1] and the more exactly the axes of the light-cones incident upon the grating are made to coincide, the smaller the deviations from spectrogram to spectrogram become.

A comparison of four observations with the grating spectrograph from April to June, 1921, and the mean of measures in 1917 for the 13 solar lines common to all plates gives for the fractional part of the mean wave-length the following:

		1917, Mean	1921, April 21	1921 April 29	1921 May 1	1921 June 1
13 lines...........	λ 4337–λ 4401	0.5545	0.5552	0.5550	0.5565	0.5554

The three April-June observations for the other common lines also show good agreement in the fractional part.

		1921, April 21	April 29	May 1
17 lines...............	λ 4489–λ 4607	0.5574	0.5594	0.5593
15 lines...............	λ 4615–λ 4688	0.3942	0.3941	0.3945
20 lines...............	λ 4707–λ 4784	0.5217	0.5217	0.5216

These wave-lengths are determined from the arc standards, not by measuring the displacements between the center of the sun and arc, but by using the arc lines as standards of reference. Five interferometer spectrograms likewise referred to arc standards, taken on August 5, 1920, at intervals from 10:44 A.M. to 5:03 P.M. P.S.T. and one taken a year later on August 8, 1921, give deviations from the mean of the six as follows for a group of 38 solar lines:

		August 5, 1920					August 8, 1921
		10:44	2:34	3:30	4:22	5:03	4:50
38 lines...	λ 5497–λ 5862	.0000	+.0011	+.0003	−.0020	+.0005	.0000 A

[1] *Observatory*, **43**, 260, 1920.

The degree of constancy of the solar wave-lengths at the center of the sun shown by these determinations is further evidenced by considering the data in Table IV from this point of view.

The wave-lengths of the oxygen lines in the table, except for the years 1920–1921, were referred to solar standards determined as above. In 1911, 1912, 1913, and 1919 the wave-lengths of the solar lines at the center of the disk were used. For the years 1920–1921 the oxygen lines were referred directly to the iron-arc standards. This completes the cycle: arc–sun–atmospheric lines–arc. The closing of the cycle is practically perfect. Unless it is assumed that continuously compensating changes in the wave-lengths of the solar and terrestrial lines occur, the agreement between the first and last sections of Table IV may be taken as a measure of the constancy in wave-length of the atmospheric lines and of the solar lines at the center of the disk, and also of the accuracy of their determination in terms of the international iron-arc standards. In 1914–1918 the results are less dependable, as the measures were made upon spectrograms of the limb and it was necessary to apply a correction for the limb-center shift, a somewhat uncertain quantity for a single observation. From an extended series the mean limb-center shift for solar lines in this spectral region is 0.007 A with a range of 0.002 to 0.010 A.

In view of the probable existence of radial convection currents on the sun the relative constancy of wave-length at the sun's center is somewhat surprising. At any given level, the radial velocity appears to reach a comparatively steady state, differing from level to level and in widely different levels even opposite in direction. For the strong lines—high-level lines—the motion is apparently downward. The great intensity of these lines at high levels is probably dependent on the increased ionization under the low pressures at the high elevations.[1] As yet too little is known of the ionizing potentials of the elements involved for a definite conclusion. For very low-level lines the motion is small but upward. At both low and high levels we appear, however, to be dealing with remarkably steady states. In this regard there is a marked difference between the center and the limb, the range of deviation from

[1] Megh Nad Saha, *Philosophical Magazine*, **40**, 472, 1920.

plate to plate at the limb being four- to five-fold that at the center. Though the causes of the motions in the solar and terrestrial atmospheres are different, there may be common characters in the general atmospheric movements due to the similar damping influences. In both cases centrifugal velocities are opposed by gravitation, very powerfully so in the sun, while tangential movements are damped by the smaller forces of friction. Thus through a kind of selective action the tangential components would tend to persist and the radial components to be quenched, so that in both the solar and terrestrial atmospheres the prevalent atmospheric flows would tend to be tangential. In the sun the opposing effects of light-pressure and gravitation may have influence in producing the steady state at any given level, the different levels of the absorbing centers being again a selective result as the equilibrium between light-pressure and gravitation depends upon the ratio between the mass and diameter of the centers.

From these observations it is evident that differences between solar wave-lengths at the sun's center and in terrestrial sources may be determined with an accuracy comparable to that for the terrestrial lines. Upon the magnitude of these differences, their variation over the solar disk, from element to element, from line to line, and from wave-length to wave-length, rests the possibility of disentangling the causes of the displacement of solar lines, a formidable but far from hopeless undertaking.

MOUNT WILSON OBSERVATORY
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THE WAVE-LENGTH IN ASTRONOMICAL INTERFEROMETER MEASUREMENTS[1]

By J. A. ANDERSON

ABSTRACT

Effective wave-length in measurements with astronomical interferometer.—The separation of double stars and the diameters of stellar disks are measured by the interferometer in terms of an effective wave-length λ which is such that if the stellar object emitted monochromatic light of wave-length λ the setting of the interferometer would be that actually found. The accuracy of setting obtainable in these measurements is such that in each case λ should be known to a few tenths of 1 per cent. *For any star,* the normal effective wave-length may be computed from the effective temperature corresponding to its spectral type, by combining the Planck radiation law with the mean transmission curve for the atmosphere and with the A.I.S. visibility-curve for the normal eye. The corrections required for variations of atmospheric absorption and of eye sensitivity from the assumed values may best be obtained experimentally by observations on sunlight, since in any case the correction is equal to the difference between the observed and computed effective wave-lengths for sunlight under the same conditions. The method and apparatus used in the *determination of the effective wave-length for sunlight* are described, and the results of some observations on the *variation of λ with zenith distance and atmospheric conditions* are given. On Mount Wilson the values found vary from 5660 A at sunrise and sunset to 5510 A for zenith distances up to 60°; at Pasadena the variation with zenith angle was more rapid. The wave-length was greater on cloudy days and least after a rain.

Astronomical interferometer.—The distribution of intensity in the interference patterns, for both circular and rectangular apertures, is fully discussed, and in the particular case of a double star is given mathematically for all points of the focal plane, for all orientations of the apertures. *The effect of the size of the apertures on the setting* is nil when rotation is used and also when the setting is made for minimum visibility of the fringes; but when the apertures are separated until minimum intensity at the center is obtained, it is found both theoretically and experimentally that for apertures of width a and separation D, the effective separation is very closely equal to $D/[1+K\,(a/D)^2]$ for D/a greater than 3.5, where K is 0.223 instead of 0.765 as given by Hamy.

The interferometer as used in astronomical measurements consists of two apertures alike in shape and size, placed either in front of the objective or in the converging beam a short distance in front of the focal plane. The idea of using a telescope in this manner appears to have originated with Fizeau,[2] and the first actual trials were made by Stephan.[3] Michelson[4] carried both the theory and applications of the method considerably farther than his predecessors; he gave an analytical treatment for the case of a pair of narrow slits, and applied it to the determination of the

[1] *Contributions from the Mount Wilson Observatory,* No. 222.

[2] *Comptes Rendus,* **66,** 934, 1868. [3] *Ibid.,* **78,** 1008, 1874.

[4] *Philosophical Magazine,* **30,** 1, 1890.

diameter of Jupiter's satellites. Hamy[1] treated the case of narrow slits, and also that of rectangular apertures whose width cannot be neglected in comparison to their separation, and applied his results to a measurement of the diameter of Jupiter's satellites and of some of the brighter asteroids. Comstock[2] used the method for a determination of the effective wave-length of star light. The work of the past two years is so recent that references to it need not be given here in detail.

The purpose of the present paper is to call attention to the necessity of knowing quite accurately the effective wave-length of the source which is studied; to describe and illustrate a method for determining this quantity quickly and accurately; and to discuss in some detail the effect of using apertures of different shapes and sizes.

Consider first the determination of the diameter of a star, assumed to appear as a uniformly luminous circular disk. If the apertures are narrow in comparison with their separation, the angular diameter is given by

$$\beta=\frac{1.22\lambda}{D} \tag{1}$$

where β is the angular diameter, D the distance between the centers of the apertures corresponding to the first vanishing of the interference fringes as the distance is increased, and λ the wave-length of light. The two quantities which determine β are λ and D. Experience indicates that the probable error in the determination of D under good conditions is of the order of 1 per cent; and accordingly if the uncertainty of the true value of λ exceeds a few tenths of 1 per cent, the probable error of β will be larger than that of D.

Again, consider a double star whose angular separation is to be measured. This is given by

$$\alpha=\frac{\lambda}{2D} \tag{2}$$

where α is the angular separation, the other quantities being the same as in (1). In this case, under good conditions, the uncertainty in the value of D is considerably less than 1 per cent.

[1] *Bulletin Astronomique*, **16**, 257, 1899.

[2] *Astrophysical Journal*, **5**, 26, 1897.

Other cases might be given, but always the quantity to be measured will be expressed in the form $K\lambda$, where K is determined by the actual observation, while λ in general must be found by some other means, and since λ is always a simple multiplier, it is necessary or at least desirable to know its value with a probable error considerably smaller than that involved in K.

At present all observations with the interferometer are made visually, and hence the value of λ will fall somewhere near the middle of the visibility curve of the eye, at least for sources emitting white light; or, we may say that λ will have a value lying somewhere within the range 5400 A to 6000 A. It follows from what has just been said that, if possible, λ should be known for a given source within 10 A to 20 A.

Definition of the effective visual wave-length.—*a*) Let an artificial double star be illuminated with the white light whose effective visual wave-length is required; let this double star be viewed with an interferometer, and the latter adjusted so that the interference fringes just disappear according to equation (2). Leaving the interferometer unchanged we now illuminate the double star with the monochromatic light whose known wave-length can be varied at will. The value of λ which just makes the fringes disappear is defined as the effective visual wave-length of the white source in question.

b) Analytically[1] the definition may be given as follows: Take rectangular co-ordinates in the focal plane with the origin on the axis of the telescope. Let the geometrical images of the components of a double star fall at $(+c, 0)$ and $(-c, 0)$. Let the apertures be narrow slits separated by a distance D, and the focal length of the objective be F.

If $I(\lambda)d\lambda$ be the visual effect of the radiation of the source lying between the limits λ and $\lambda+d\lambda$, the intensity along the x-axis is given by

$$J=\int_0^\infty I(\lambda)d\lambda\left[\cos^2\frac{\pi D(x-c)}{F\lambda}+\cos^2\frac{\pi D(x+c)}{F\lambda}\right] \tag{3}$$

[1] This statement was first suggested to the writer in a letter from Dr. C. M. Sparrow.

When D is very small, J has a maximum at $x=0$. As D increases, J diminishes for $x=0$, reaching a value of zero when D is somewhere near $F\lambda_0/2c$ where λ_0 is an arbitrary value of λ near the middle of the range for which $I(\lambda)$ has an appreciable value. Both for D small and for D having a value near $F\lambda_0/2c$, J oscillates as x is varied; but for a value of D in the neighborhood of $F\lambda_0/4c$, these oscillations become very small, and in particular at $x=0$, $J(x)$ will have a stationary value for some particular value of D. The condition for this is

$$\frac{\partial^2 J}{\partial D^2}=0; \quad \text{for } x=0 \tag{4}$$

Solving (4) we have $D=F\lambda_0'/4c$, where λ_0' is the effective visual wave-length required.

For a source such as a given star, $I(\lambda)$ is defined as follows: Let $E(\lambda)$ represent the energy-curve of the star, as determined outside our atmosphere; let $T(\lambda)$ be the transmission-curve of the atmosphere at the time and place of the observation; and let $V(\lambda)$ be the visibility-curve of the eye of the observer; then

$$I(\lambda)=E(\lambda)T(\lambda)V(\lambda).$$

Some questions now naturally arise, among which the following are important enough to state:

1. If in (*a*), instead of using an artificial double star, we use an artificial star disk, would the same value of λ be obtained?

2. If the components of a double star are of different spectral types, or, what amounts to the same thing, if the effective wave-length of the components is not the same, what wave-length is to be used in calculating the angular separation from the data given by an observation with the interferometer?

3. In case of a star disk darkened toward the limb, if this darkening is accompanied by a color change, what value of λ is to be used?

It is important to remember that when the observations are made visually, different observers will not in general find the same effective wave-length for a given source. In (*b*) above this is provided for by the factor $V(\lambda)$, which may differ slightly for different observers. In (*a*) it is automatically provided for by reason of

the fact that the same observer is supposed to make both the adjustments. Again, since $T(\lambda)$ enters as a factor in $I(\lambda)$, an observer will not obtain the same value of λ for a given source under different conditions of observations as to time and place.

Only a beginning has thus far been made in the application of the interferometer to astronomical measurements, and it is too early to predict what its future may be; yet it will undoubtedly be valuable in certain restricted fields; for example, in the direct determination of star diameters and in the measurement of very close double stars of particular types. No doubt it will be applied successfully also in other fields. Now, as has already been pointed out, the result of any interferometer measure[1] will be expressed in the form

$$\theta = K\lambda \tag{5}$$

where K is given by the observation, and λ depends upon the object observed. If λ did not depend upon local conditions such as the observer, the time and place of the observation, and the instruments employed, the reduction would be a relatively simple matter; for this case the value of λ could be found, once and for all, for stars of all spectral types. It is clear, however, from what has already been said, that this may involve errors very much larger than the errors of observation. On the other hand, it will obviously be quite impracticable for each observer to determine independently the value of λ for every observation he makes. The following plan will, it is hoped, remove most of the difficulties. We will leave out of account objects such as the planetary nebulae, that is, objects having discontinuous spectra, for in such cases there is no difficulty in determining the value of λ with sufficient accuracy. Consider then only objects like the great majority of the stars, having continuous spectra, more or less similar to spectra of black bodies at various temperatures. The fact that no star radiates exactly like a black body is of little importance, because the black-body temperature will be used merely as an auxilliary in the general scheme. The black-body temperature of stars has been determined for all or nearly all the brighter stars, and these include nearly all of the spectral types that it will be necessary to consider.

[1] The position-angle of a double star is an exception.

Using Planck's law, the mean transmission-curve of the atmosphere, and the adopted *A.I.S.* visibility-curve of radiation, we may compute by equation (4) the value of λ_0' for a few temperatures from 3000° to 20,000° absolute. A curve passed through the computed points will enable one to read off λ_0' for any temperature within the range. The absolute value of the wave-lengths thus derived may not be correct, but this does not matter, for their relative values will be sensibly right. To illustrate the use of such a curve 'n practice, suppose that an observer has made an interferometer measure of a star of spectral type F0; assume also that he has made a determination of the effective wave-length of sunlight by the method given later in this paper. From tables giving the effective temperatures of stars of different spectral types, he reads off the temperature for an F0 star and for a star of the same spectral type as the sun. Let these temperatures be, respectively, T_1 and T_2. From the curve the corresponding wave-lengths are λ_1 and λ_2. If now the measured wave-length of sunlight is λ_0, the proper wave-length for the F0 star will be

$$\lambda = \lambda_0 + \lambda_1 - \lambda_2 \tag{6}$$

Of course, as the determination of λ_0 is necessarily made in daytime while the observation of the star is made at night, it is probable that the atmospheric conditions will have changed in the interval. But as different determinations of λ_0 will soon show about what correction is required for variations in observing conditions, this difficulty is not very serious.

Experimental determination of effective wave-length.—In Figure 1 let A be an artificial double star, consisting of two small pinholes

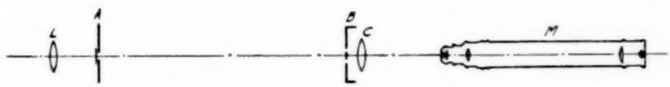


FIG. 1

whose linear separation is δ, illuminated by white light such as sunlight. At B are two circular apertures separated by the distance D, so mounted that they can be rotated about the axis of the telescope C. Observations are made through the compound

microscope M. The distance A to B will be denoted by L. The angular separation of the pinholes as viewed from B is δ/L. The value of D for the first disappearance of the interference fringes, if the line joining the pinholes is parallel to that joining the apertures at B, is, by equation (2),

$$\frac{\delta}{L}=\frac{\lambda}{2D}$$

or, if the two lines make an angle θ with each other,

$$\frac{\delta}{L}=\frac{\lambda}{2D\cos\theta} \tag{7}$$

All the quantities in the equation except λ are directly measurable, and hence λ can be determined.

In order to form some idea of the accuracy to be expected in this determination, we will now examine in detail the measurement of each of the quantities entering into equation (7).

In the present experiments δ was of the order of 0.6 mm. The two pinholes were made in tinfoil, as follows: A fine needle was selected and mounted in a mechanics' pin-chuck. A small piece of tinfoil was smoothed out on a flat, smooth piece of soft metal such as brass or aluminum. A small round hole was made by resting the needle point on the tinfoil, the needle and chuck being vertical, and then carefully turning the chuck a few times. By always taking care to have the pressure about the same, holes of very nearly the same diameter may be made, and if care is used in rotating the chuck and in lifting the needle after the operation, the holes will be quite perfectly round. The diameter of the pinholes used was between 0.035 and 0.050 mm. Under a microscope the distance between such a pair of openings can be determined with a probable error of about 0.0002 mm. Since δ is about 0.6 mm, this corresponds to a probable error of the order of 1 in 3000. Before measurement the tinfoil is mounted on a metal plate through which a hole about 2 mm in diameter has been drilled, the apertures in the tinfoil being directly over this hole.

Two sources of error are to be noted in using such an arrangement as an artificial double star:

1. The plane of the apertures will not in general be exactly perpendicular to the line of sight from the observing telescope, but will make an angle ϕ with this line. The apparent separation will then be not δ but $\delta \sin \phi$. With ordinary care in mounting the tinfoil and adjusting the plate, ϕ should not differ from $90°$ more than a degree, which would at most cause an error of 15 parts in 100,000, which is negligible.

2. When an image of the sun is formed on the apertures, the tinfoil becomes heated somewhat, causing δ to increase. The rise in temperature to be expected may be roughly estimated. The image-forming lens has an area of 16 cm^2, and its focal length was 1 meter, so the diameter of the image was roughly 1 cm. Of this $\frac{1}{25}$ fell on the tinfoil, which is equal to the amount of sunlight falling on $\frac{16}{25}$ cm^2, or 1 calorie per minute. Assuming that the tinfoil reflects 80 per cent of this, we have $\frac{1}{5}$ calorie per minute taken up by the tinfoil or 0.003 calorie per second. When equilibrium is established, the greater part of this energy passes by conduction from the tinfoil to the brass plate, which is so large that its temperature never rose appreciably above room temperature. A temperature gradient of $3°$ per centimeter in the tinfoil would more than suffice to carry this amount of energy to the brass plate—and hence we may conclude that, within a very few degrees, the tinfoil was always at room temperature, and that the error due to thermal expansion was less than 1 in 10,000, which is negligible.

The value of D was usually between 6 and 7 mm. The apertures were round holes made with a drill in a brass plate; they were always sensibly equal in diameter, and the determination of their distance apart could always be made with an error less than 1 part in 5000. Various diameters were used, from about 1 mm up to 2.5 mm. The effect of using apertures of different sizes will be discussed more fully below.

The value of L was about 900 centimeters, and with a good steel tape this distance could always be measured to a millimeter.

It follows that, as far as the quantities δ, D, and L are concerned, the sum of the errors made in their evaluation should never affect the result by as much as 1 part in 1000; and, since the wave-

length of the light used was near 5500 A, the uncertainty arising from the instrumental constants is not more than 5 A.

It remains now to discuss the value of θ, which is the only quantity directly measured in an actual experiment. In order to make the discussion of θ intelligible it is necessary to describe and explain the appearance of the diffraction pattern and its system of interference fringes in some detail.

Consider the distribution of intensity in the focal plane, due to a single distant point source, when the objective is covered by a screen perforated by a single circular aperture of radius r. Denote the focal length by F, and let the origin of rectangular co-ordinates coincide with the optical axis. The intensity is given by

$$I=K\frac{4J_1^2\dfrac{2\pi r\sqrt{x^2+y^2}}{F\lambda}}{\left(\dfrac{2\pi r\sqrt{x^2+y^2}}{F\lambda}\right)^2} \tag{8}$$

where J_1 is a Bessel function of the first order.

This well-known expression states that at $x=0$, $y=0$, the intensity is a maximum and equal to K; as we move away from the origin, the intensity diminishes at first very slowly, then more rapidly, and reaches a value zero, when the argument of J_1 is equal to about $219°33'$. For larger values of the argument the intensity increases, reaching a second maximum whose value is $0.0175\,K$ near $292°$, and a second zero value near $401°$, etc. This is, of course, simply the ordinary diffraction pattern of a circular aperture, or the so-called spurious star disk. In ordinary telescopic observations with relatively large apertures the entire disk and diffraction rings are apparently very small; but in interferometer work, on account of the small apertures and the very high magnification employed, the pattern may cover the larger part of the field of view.

The appearance is independent of the location of the aperture, as long as this falls entirely within the objective. Hence we will assume that it is placed so that its center is at a distance $D/2$ from the axis. Now let an equal aperture be uncovered, at the same

distance from, but on the other side of, the axis. This second aperture alone would give exactly the same intensity as the first, namely that expressed by equation (8); but if we use both apertures together, the result will be the sum, not of the intensities as given by (8), but of the amplitudes formed by taking the square root of the right-hand side of (8), with proper attention to the relative phases of the two disturbances.

Let the apertures be so placed that the line joining their centers is parallel to the x-axis; in this case the amplitudes and phases will be equal at $x=1$. At $x=\pm F\lambda/2D$ the phases will differ by 180°, and since the amplitudes are equal, the resultant amplitude is zero. At $x=\pm F\lambda/D$, the phases differ by 360°, that is, they are in phase, and the amplitude will be the sum of the two, and so on.

It follows that the resultant intensity is now given by

$$I=I_0 \frac{4J_1^2 \dfrac{2\pi r\sqrt{x^2+y^2}}{F\lambda}}{\left(\dfrac{2\pi r\sqrt{x^2+y^2}}{F\lambda}\right)^2} \cos^2 \frac{\pi Dx}{F\lambda} \tag{9}$$

I_0 being the intensity at the origin.

Hence the general outline of the pattern is the same as that given by a single aperture; we still have the central disk surrounded by diffraction rings; but the effect of using two apertures is to break up the disk and rings into a series of equally spaced interference fringes parallel to the y-axis—that is, perpendicular to the line joining the apertures. Along a line parallel to the y-axis the distribution of relative intensity is exactly the same as that in the pattern given by a single aperture; parallel to the x-axis we have this intensity multiplied by the factor $\cos^2 \pi Dx/F\lambda$.

Figure 2 gives a graphical representation of the intensity along the x-axis. The dotted curve is the intensity due to a single aperture, multiplied by the factor 4.

If we retain the apertures, but use two point-sources separated by an angular distance α, there will be two such patterns, whose centers are separated by the distance $2c=F\alpha$. If the centers fall

at $(-c, 0)$ and $(+c, 0)$, respectively, the expression for the intensity becomes

$$I=I_0\left\{\frac{4J_1^2\left(\frac{2\pi r\sqrt{(x-c)^2+y^2}}{F\lambda}\right)}{\left(\frac{2\pi r\sqrt{(x-c)^2+y^2}}{F\lambda}\right)^2}\cos^2\frac{\pi D(x-c)}{F\lambda}\right.$$
$$\left.+\frac{4J_1^2\left(\frac{2\pi r\sqrt{(x+c)^2+y^2}}{F\lambda}\right)}{\left(\frac{2\pi r\sqrt{(x+c)^2+y^2}}{F\lambda}\right)^2}\cos^2\frac{\pi D(x+c)}{F\lambda}\right\} \qquad (10)$$

If the line joining the apertures makes an angle θ with the x-axis, then, letting $m=\tan\theta$, the expression for the intensity will be that given by equation (10), except that we write $x+my$ instead of x in the cosine factors. Figure 3 is a sketch showing the pattern for such a double point-source, the line joining the apertures making the angle θ with x-axis. The centers of the patterns are at P and P' respectively; the pattern centered at P' is shown in dotted lines. The straight lines represent the centers of the bright fringes, AA being the central fringe for P', $A'A'$ that for P'. The angle θ has been so chosen that the fringes due to P' are just midway between those due to P. If θ is diminished, the similarly lettered fringes will approach each other and the visibility will be increased; and similarly, if θ is increased, AA will approach $B'B'$, etc., and the visibility will again increase. The position represented is that of minimum visibility.

Remembering that the intensity in a pattern diminishes from the center outward, it is clear that at all points on the y-axis the intensity of one pattern is the same as that of the other, except for the changes due to the interference, and since the maxima in one pattern fall on the minima of the other, the result is very nearly uniform intensity along this line. To the right of the y-axis the intensity of P' is greater than that of P, and hence the maxima of the P' pattern become more and more prominent as we go to the right. To the left of the y-axis the P pattern is stronger and hence its maxima become increasingly prominent in this direction. The observer will therefore see two sets of fringes meeting just out of

step near and along the y-axis. (a) If θ is decreased a certain amount, so that the distance between AA and $A'A'$ becomes smaller than the adjacent intervals, the corresponding bright fringe will start at A' on the right and run in a sensibly straight line to A on the left, crossing the y-axis exactly at the origin. (b) If θ is increased, we shall have a dark fringe running from A on the right in a nearly straight line to A' on the left, also crossing the y-axis at the origin. The change in θ necessary to pass from condition (a) to condition (b) depends upon the number of fringes in the pattern as here represented, being the smaller, the larger the number of fringes. For a case such as that in Figure 3, with 6 fringes across the central disk, the total required change in θ is about 5°. With 12 to 15 fringes it will be from 2° to 3°. It is evident, therefore, that a setting for the exact bisection can be made quite accurately—in general to a few tenths of a degree.

We can now estimate the probable error in the determination of θ. In one rotation of the aperture plate there are four values of θ which give the appearance just described, and a "set of observations" in the present experiments has consisted of five complete rotations. As a typical example we may take the measures made on Mount Wilson on June 18, 1921. Fourteen sets of observations were made of which only eleven were complete. The probable error of each set was calculated in the usual way, and the sum of these probable errors found to be $0\overset{\circ}{.}48$, or an average of about $0\overset{\circ}{.}035$ for a set. Taking $\theta = 45^\circ$, the percentage error of $\cos\theta$ is 0.05 per cent or 1 part in 2000.

Hence the experimental method should be capable of giving λ to at least 1 part in 1000, or to about 5 A.

Effect of the size of the apertures.—Hamy[1] derived a formula for the correction to be applied to observations made with apertures of finite width. The apertures were assumed to be rectangular, of width a and separation D: The source was assumed to be a uniformly luminous circular disk of angular diameter β. The fringes will disappear near the center of the pattern when

$$D = \frac{1.22\,\lambda}{\beta}\left\{1 + 0.765\left(\frac{a}{D}\right)^2 \;.\;.\;.\;.\right\} .$$

[1] *Bulletin Astronomique*, **16**, 257, 1899.

No other cases were treated. It would seem reasonable to assume, however, that if the source is a double star of angular separation α, the corresponding formula would be

$$D=\frac{\lambda}{2\alpha}\left\{1+0.765\left(\frac{a}{D}\right)^2\ \ldots\ldots\right\}$$

since the correction must depend upon some property of the observing instrument. For circular apertures, a further modification would no doubt be required, and we may reasonably assume that this would be the substitution of $a'/1.22$ for a where a' is the diameter of the circular aperture.

Observations were made on the artificial double star with circular apertures of various sizes, using the method of rotating aperture plate. These gave the *same* value of θ for all sizes of apertures, well within the errors of observation, thus indicating that for this method of observing and with this kind of a source no correction whatever is required. This result was so surprising that it seemed worth while to make a theoretical examination of the problem, using a method somewhat different from that employed by Hamy.

The method adopted, while being very simple, has the advantage of giving a clear physical picture of the phenomenon so that the need for a correction, if required, is made evident.

Let us consider rectangular apertures having a width a, and a length b, and let their centers be separated by a distance D. When the line joining the apertures is parallel to the x-axis the intensity in the focal plane due to a distant point-source is

$$I=I_0\frac{\sin^2\dfrac{\pi ax}{F\lambda}}{\left(\dfrac{\pi ax}{F\lambda}\right)^2}\frac{\sin^2\dfrac{\pi by}{F\lambda}}{\left(\dfrac{\pi by}{F\lambda}\right)^2}\cos^2\frac{\pi Dx}{F\lambda} \tag{11}$$

If a double point-source is used, the angular separation being α, we have, writing $2c=F\alpha$

$$I=I_0'\left\{\frac{\sin^2\dfrac{\pi a(x-c)}{F\lambda}}{\left(\dfrac{\pi a(x-c)}{F\lambda}\right)^2}\frac{\sin^2\dfrac{\pi by}{F\lambda}}{\left(\dfrac{\pi by}{F\lambda}\right)^2}\cos^2\frac{\pi D(x-c)}{F\lambda}\right.$$

$$\left.+\frac{\sin^2\dfrac{\pi a(x+c)}{F\lambda}}{\left(\dfrac{\pi a(x+c)}{F\lambda}\right)^2}\frac{\sin^2\dfrac{\pi by}{F\lambda}}{\left(\dfrac{\pi by}{F\lambda}\right)^2}\cos^2\frac{\pi D(x+c)}{F\lambda}\right\} \quad (12)$$

If circular apertures of radius r be employed, we have equation (10) instead of the foregoing.

These expressions hold for monochromatic light of wave-length λ, and hence by definition (a) they are also valid for white light of visual effective wave-length λ. If we limit ourselves to the phenomena on the x-axis, which in no way restricts the generality of the treatment, we can place the factors involving y equal to 1 in equation (11) and write x for $\sqrt{x^2+y^2}$ in equation (10). Writing $D/a=n$, equation (12) may be put in the form

$$I=I_0'\left\{\frac{\sin^2\dfrac{z-c'}{n}}{\left(\dfrac{z-c'}{n}\right)^2}\cos^2(z-c')+\frac{\sin^2\dfrac{z+c'}{n}}{\left(\dfrac{z+c'}{n}\right)^2}\cos^2(z+c')\right\} \quad (12a)$$

in which $c'=\pi Dc/F\lambda$.

When n is very large, the intensity near the center of the pattern is given very nearly, by

$$I=I_0'\{\cos^2(z-c')+\cos^2(z+c')\}.$$

The visibility will be zero when $z-c'=z+c'-\frac{\pi}{2}$, or, $2c'=\frac{\pi}{2}$, or, applied to equation (12), we find that for a small in comparison to D, the fringes will vanish when

$$\frac{2\pi Dc}{F\lambda}=\frac{\pi}{2}$$

whence $2c=F\lambda/2D$, or, since $2c/F=\alpha$,

$$\alpha=\frac{\lambda}{2D}$$

which is the ordinary formula, equation (2).

In order to form a clear idea of what happens when a is increased, let us refer to Figure 2, and for this purpose we will assume that the dotted curve represents

$$\frac{\sin^2 \frac{z-c'}{n}}{\left(\frac{z-c'}{n}\right)^2}$$

instead of

$$\frac{4J_1^2\left(\frac{2\pi rx}{F\lambda}\right)}{\left(\frac{2\pi rx}{F\lambda}\right)^2}$$

which really makes very little difference.

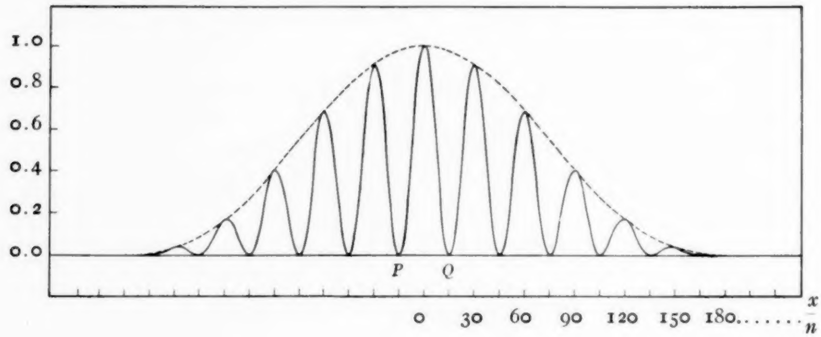


FIG. 2.—Dotted curve; $y = \frac{\sin^2 \frac{x}{n}}{\left(\frac{x}{n}\right)^2}$ $(n=6)$

Full curve: $y = \frac{\sin^2 \frac{x}{n}}{\left(\frac{x}{n}\right)^2} \cos^2 x$

With this understanding, we have then $2n$-loops of the inscribed curve in the central loop of the dotted curve. If n is very large, or a very small, the loops of the inscribed curve near the origin all have very nearly the same height, the dotted curve approaching a straight line parallel to the x-axis, and the inscribed curve approaching a true cosine-square curve more and more closely as

n is increased. If, under these conditions, we superpose two patterns displaced in the x direction by one-quarter of a period, the maxima of one will fall upon the minima of the other, and the sum will be a constant, which leads directly to equation (2).

When n is not large, the inscribed curve may be regarded as a distorted cosine-square curve, the distortion becoming more and more pronounced the more the dotted curve deviates from a straight line parallel to the axis. Since with white light the fringes are observable only near the center of the pattern, it follows that we need consider only the loops of the inscribed curve which are near the origin, and when n is small we have to confine our attention to the central fringe only, or to the portion of Figure 2 between the points P and Q. Let the ordinate at $x=0$ be taken as unity, and expressing x in degrees, we have for the undistorted curve ($n=\infty$) the value 0.500 at $\pm 45°$ and the value 0 at 90°. For the distorted curve we have unity at $x=0$, and zero at 90°, but a value less than 0.500 at 45°. Hence if we superpose two such curves, 90° apart, there will be a minimum at 45°, or at the point midway between them, which will disappear only if their distance apart is made less than 90°. This makes it clear why equation (2) requires modification for small values of n, at least if we regard only the center of the pattern. It is necessary to remember, however, that when the minimum at 45° for the superposed curves disappears, the minimum at 135° has been made more prominent, and accordingly the "correction" that we employ to remove the minimum at 45° is really somewhat too large.

In order to calculate the angular displacement necessary to fill up the minimum at the center of the superposed curves, we need only equate the second derivative of (12) with respect to c, equate it to zero, and solve for c on the assumption that $x=0$. The factor in y is of course placed equal to unity. Table I gives the results of this calculation for a few values of n.

Here we have written the formula applicable to double stars in the form

$$D=\frac{\lambda}{2a}(1+K)$$

where K according to Hamy is $0.765\left(\frac{a}{D}\right)^2$.

Experimental verification of the results given in Table I.—In Table I we have applied Hamy's formula to a case which was not considered in his investigation, namely that of double stars. The difference between our calculated values and those given by

TABLE I

n	c	K(Calculated)	K by Hamy's Formula	Ratio $\frac{K\text{(Hamy)}}{K\text{(Calc.)}}$
∞	45°000	0	0	....
11.5	44.9234	0.0017	0.00577	3.395
5.75	44.6938	0.0068	0.02314	3.403
4.60	44.5275	0.0106	0.03615	3.411
3.45	44.1721	0.0187	0.06427	3.437

Hamy's formula as here applied is so great that it seemed worth while to test both the case of double stars and that of a star disk experimentally, to see if there is really any difference between the two.

The experimental arrangement was as follows: A screen having a horizontal slot 2 mm in width was placed in front of the objective of the observing telescope. A plate was arranged to slide in a vertical direction directly in front of and in contact with this screen. This plate had two parallel slots 5 mm apart, each tapering from $\frac{1}{2}$ mm in width at one end to $2\frac{1}{2}$ mm at the other end, the length being 55 mm. This arrangement provided two nearly rectangular openings in front of the objective, at a constant distance apart; by sliding the plate up and down the width of each opening could be varied continuously from $\frac{1}{2}$ mm to $2\frac{1}{2}$ mm, or, what amounts to the same thing, n could be varied from $n=2$ to $n=10$ in a continuous manner.

Observations were made both on an artificial double star, and on an artificial star disk. The disk was illuminated with approximately monochromatic green light, in order to avoid the variations in color which are troublesome with white light, especially when the visibility of the fringes is low.

The observations were made by finding the distance between the observing telescope and the source which gave the minimum

visibility for $n=10$, and also the distance at which the central minimum for $n=2$ just vanished. Since the results were the same as far as the correction factor is concerned, whether a star disk or a double star was used, we will record only the observations made on the star disk:

Wave-length of green light (mean) 5400 A.
Diameter of artificial star disk (δ) 0.650 mm.
Distance between centers of rectangular apertures $(D)=5.0$ mm.
Minimum visibility $n=10$, at 492 cm ± 2 cm.
Minimum in central fringe just disappears $(n=2)$ at 478 *cm* ± 2 cm.

Using $L=D\delta/1.22\,\lambda$ for the distance at which fringes should vanish with $n=\infty$ and substituting, we get $L=493.3$ cm, which, considering the uncertainty in the value of λ used, is sufficiently near 492 cm.

From this we find an experimental value of K for $n=2$, namely, $\frac{492}{478}-1$ or $\frac{493.3}{478}-1$, according to whether we use the observed or calculated value of L_∞. The first gives $K=0.0293$, the second $K=0.0320$, while by Hamy's formula, which is strictly applicable to this case, K should equal 0.19125. An extrapolation of the values in Table I to $n=2$ gives a value of K slightly larger than 0.03, which is in good agreement with the observations. We must conclude that the formula given by Hamy is incorrect, and that if the observations are made directly on the vanishing of the minimum in the central fringe, the values given in Table I are correct. It must be borne in mind, however, that if the observations are made simply on the minimum visibility of the pattern as a whole, which on account of seeing, will generally be the case in astronomical work, the simple formulae (1) and (2) are to be used without any corrections. It has already been shown (see p. 64) that, with the method of rotating apertures, no correction to the simple formula is required.

Determination of the effective wave-length of sunlight.—We will first consider some general questions which bear directly on the problem. It may be assumed that the quality of sunlight outside of our atmosphere is constant or very nearly so. In passing through the atmosphere it is modified by selective absorption and

scattering to an extent which depends both on the thickness of the air layer and upon its character, such as water-vapor content, smoke, haze, etc. Most of these factors will reduce the intensity of the short wave-lengths more than that of the red end of the spectrum. Water-vapor is, however, an exception. It is very transparent to the short wave-lengths, while absorption begins somewhat below 6000 A and increases rapidly toward the red. Since its absorption lies almost entirely on the red side of the maximum of the sensitivity curve of the human eye, it appears that the

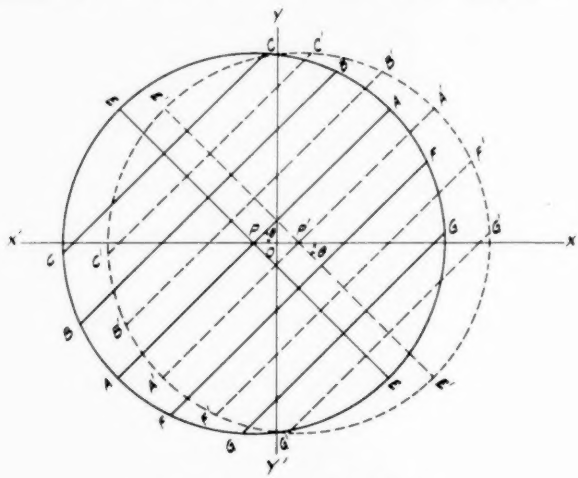
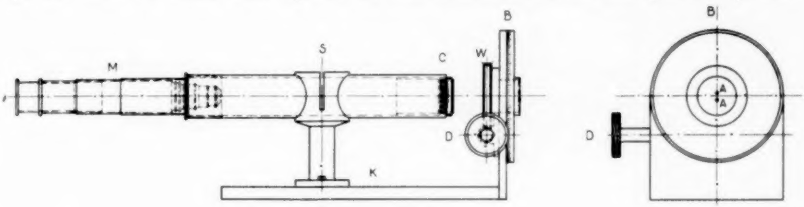


FIG. 3.—Pattern for double star

effect of water-vapor by itself is to move the value of the effective wave-length toward the violet. Dust, smoke, haze, and increased thickness of the atmosphere due to large zenith distances might be expected to shift the wave-length to the red. A priori, then, a given observer would not expect to find a constant value for the wave-length.

Apparatus.—Figure 1 gives a schematic diagram of the apparatus, and Figure 4 is a drawing of the observing telescope and aperture plate. The lens *C* is a theodolite objective of 22 mm aperture and of about 20 cm focus. The miscroscope consists of an 8-mm objective and a 7.5-power eyepiece, giving a linear magnification of about 125×. The circle *B* is graduated in degrees

and can be read easily to 0°1. It carries the aperture plate *AA*, and is rotated by a worm *DW*, so arranged that one revolution of the worm gives a rotation of 10°. The whole instrument is mounted on a solid brass base plate, *K*, which is easily clamped to a table or other convenient support. The slot *S* was designed to hold a plane parallel quartz plate, cut at 45° to the optic axis. Viewed through this plate, a single distant pinhole appears as an artificial double star, whose angular separation depends only on the thickness of the quartz plate. A slight difficulty arises in the use of such a quartz plate, on account of the fact that one of the emergent beams of light is colored, due to the dispersion of the quartz. A great deal of time was spent in the calibration of the plate by the



a *b*

FIG. 4.—Laboratory apparatus
a. Side view. *b.* End view

use of monochromatic light, and a few observations were made using it in conjunction with the 100-inch Hooker telescope on some of the brighter stars, but the results were not entirely satisfactory. When the quartz plate is not used the slot *S* is covered by a suitable cap. In the determination of the wave-length of sunlight the source was always a pair of pinholes made and mounted as described above; the sunlight was reflected into the quartz projection lens *L*, Figure 1, by a two-mirror coelostat, the mirrors being freshly silvered and polished every two or three weeks.

In Tables II and III are collected the observations which have been made with the apparatus shown in Figure 4. Table II gives the observations made on Mount Wilson, June 15 to June 18, while in Table III are collected all the miscellaneous observations made under all sorts of conditions of sky in Pasadena. The first two columns give the date and hour of the observation. Since each complete observation required about 10 minutes, the time given

refers to the middle of the observation. The third column gives the zenith distance of the sun to the nearest degree at the time. The fourth column gives the number of settings in each determination, which was always 20, excepting for the first observation on Mount Wilson which was stopped by fog, and for the three early observations on June 18, when the available time did not permit a complete set. The only observations calling for special comment are those of April 5 and 6 in Pasadena. These were made immediately after a rain, and the abnormally low value for the wave-length may be due to a relatively large amount of water-vapor; this, as mentioned above, would tend to displace the effective wave-length to the violet.

TABLE II

Mount Wilson Observations

Date	Mean Time	ζ	No. of Settings	λ	Remarks
June 15.....	8:35 A.M.	45°	16	(5546)	Incomplete. Fog blowing over. Image drifting badly.
June 16.....	5:14 A.M.	85	20	5631	
June 16.....	5:46 A.M.	79	20	5574	
June 16.....	6:35 A.M.	69	20	5542	
June 16.....	7:53 A.M.	53	20	(5535)	Fog coming over. Foggy remainder of day.
June 17.....	6:32 A.M.	69	20	5554	
June 17.....	7:52 A.M.	53	20	5515	
June 17.....	8:34 A.M.	45	20	5519	Faint haze.
June 17.....	9:18 A.M.	35	20	5519	
June 17.....	10:06 A.M.	25	20	5520	
June 17.....	10:49 A.M.	17	20	5515	Clouded over at 11:00 A.M.
June 17.....	4:50 P.M.	65	20	5546	Clouds. Haze as high as mountain tops. Sky blue.
June 17.....	5:22 P.M.	71	20	5579	Clouds. Haze as high as mountain tops. Sky blue.
June 17.....	6:13 P.M.	82	20	5596	Through very thin clouds. Haze.
June 17.....	6:37 P.M.	86	20	5625	Haze only.
June 18.....	4:58 A.M.	87	8	5656	Sky clear all day.
June 18.....	5:06 A.M.	86	8	5629	Sky clear all day.
June 18.....	5:07 A.M.	85	8	5619	Sky clear all day.
June 18.....	5:17 A.M.	83	20	5593	
June 18.....	5:47 A.M.	78	20	5561	
June 18.....	6:12 A.M.	73	20	5546	
June 18.....	6:44 A.M.	67	20	5528	
June 18.....	7:43 A.M.	55	20	5523	
June 18.....	8:12 A.M.	48	20	5512	
June 18.....	9:10 A.M.	37	20	5512	
June 18.....	10:11 A.M.	25	20	5515	
June 18.....	10:56 A.M.	17	20	5513	
June 18.....	11:20 A.M.	14	20	5510	
June 18.....	11:58 A.M.	11	20	5509	

TABLE III

PASADENA OBSERVATIONS

Date	Mean Time	ζ	No. of Settings	λ	Remarks
March 27...	10:11 A.M.	40	20	5537	
March 27...	11:41 A.M.	32	20	5521	
March 28...	9:05 A.M.	48	20	5568	
March 28...	9:57 A.M.	42	20	5531	
March 28...	11:03 A.M.	34	20	5515	
March 28...	2:03 P.M.	42	20	5555	
March 29...	9:19 A.M.	48	20	5556	
March 29...	9:50 A.M.	42	20	5542	
March 29...	10:29 A.M.	36	20	5522	
March 29...	11:30 A.M.	31	20	5525	
March 29...	1:10 A.M.	36	20	5559	Sky very thick.
March 30...	8:30 A.M.	56	20	5562	Sky thick toward east.
March 30...	9:10 A.M.	49	20	5537	
March 30...	9:40 A.M.	45	20	5542	
March 30...	10:03 A.M.	41	20	5547	
March 30...	10:27 A.M.	37	20	5528	Clearing rapidly.
March 30...	11:01 A.M.	33	20	5522	
March 30...	11:25 A.M.	31	20	5515	
March 30...	1:00 P.M.	34	20	5519	
March 30...	1:30 P.M.	38	20	5522	
March 30...	2:05 P.M.	43	20	5526	
March 30...	2:50 P.M.	51	20	5526	Cirrus developing rapidly.
April 2...	10:10 A.M.	28	20	5553	Some fog. Light unsteady.
April 2...	10:38 A.M.	34	20	5547	White sky, haze.
April 2...	11:10 A.M.	31	20	5524	
April 2...	11:30 A.M.	30	20	5524	
April 2...	1:35 P.M.	37	20	5527	
April 4...	8:30 P.M.	55	20	5524	After a rain. Cumulus clouds. Sky clear. Windy.
April 4...	8:56 P.M.	50	20	5527	
April 4...	9:48 P.M.	40	20	5524	
April 4...	10:42 P.M.	34	20	5533	
April 4...	11:17 P.M.	31	20	5531	Observer P.W.M.
April 5...	9:10 P.M.	46	20	5517	Thin cirrus here and there.
April 5...	10:12 P.M.	37	20	5494	Through cirrus.
April 5...	11:13 P.M.	30	20	5503	Clear.
April 6...	8:05 A.M.	59	20	5515	Clear.
April 6...	9:46 A.M.	39	20	5513	Clear.
April 6...	11:22 A.M.	29	20	5506	Clear.
June 21...	8:50 A.M.	40	20	5560	Haze obscuring mountains. White area around sun. Sky blue otherwise.
June 21...	9:26 A.M.	33	20	5545	
June 21...	10:12 A.M.	24	20	5533	
June 21...	11:05 A.M.	14	20	5524	
June 21...	11:40 A.M.	11	20	5522	

Figure 5 gives a graphical representation of the Mount Wilson results. This shows that on clear days for purposes of interferometer measurement the wave-length is sensibly constant if the

zenith distance is less than 60°, while for greater zenith distances it rises rapidly. The Pasadena observations in general show that for zenith distances less than 40° there is not much change in the

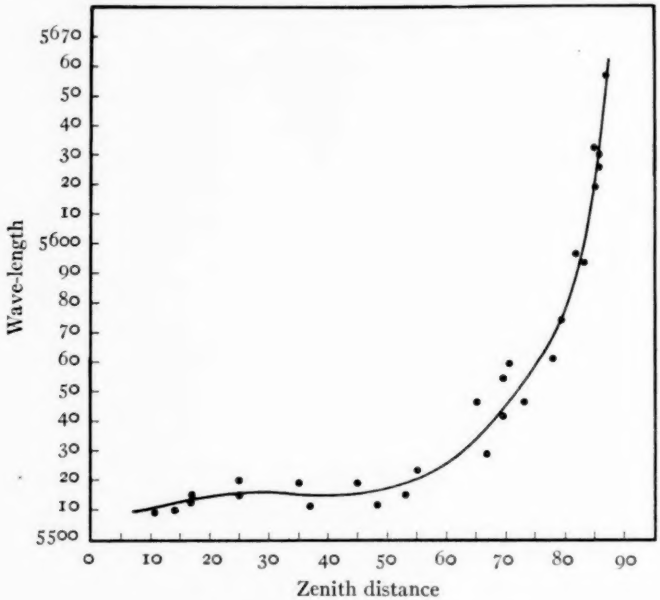


FIG. 5.—Mount Wilson observations
June 16–18, 1921

wave-length. For small zenith distances the Pasadena observations indicate a value not far from 5520, while the Mount Wilson observations indicate 5510. The difference of 10 A, though perhaps real, is so small that it may be said to be negligible as far as ordinary astronomical interferometer work is concerned.

MOUNT WILSON OBSERVATORY
October 1921

Minor Contributions and Notes

ON THE SYSTEM OF PROCYON

ABSTRACT

Radial and orbital velocities of the system of Procyon.—A series of forty-five plates made from 1908 to 1913 give for V_0, after correction for the orbital velocity, the value -3.52 ± 0.057 km/sec. Taking the parallax at 0″33 and the masses as 2.70 and 0.89, K comes out 0.60 km/sec. The sign of the inclination is assumed to be positive, but is not definitely known. In order that more accurate information as to the orbit may be obtained, it is hoped that other observatories will co-operate in measuring radial velocities during the next few years, since as favorable an opportunity will not occur again for thirty-nine years.

The most recent orbit of this well-known and interesting binary is due to Lewis Boss[1] who gave the following elements of the orbit of the faint star about the brighter one:

$$\begin{aligned} T &= 1886.5 \\ P &= 39^{\mathrm{y}}.0 \\ \Omega &= 150^\circ.7 \\ i &= \pm 14^\circ.2 \\ \omega &= 36^\circ.8 \\ a &= 4''.05 \\ e &= 0.324 \end{aligned}$$

Aitken[2] gives the masses of the two stars in terms of the sun's mass as $m = 2.70$, $m' = 0.89$.

The following values of the parallax have been published: L. Boss[1] (adopted) 0″33; Kapteyn and Weersma[3] 0″324; Adams,[4] spectroscopic, 0″347; trigonometric, 0″309. There is considerable uncertainty in the foregoing elements owing to the difficulty of the observations and the short length of the arc described by the faint companion since its discovery by Schaeberle in 1896.

As observations of radial velocity, if sufficiently numerous and differential, promise results of value, it is opportune at the present time to call attention to them and to invite the co-operation of other observatories in an attack on the problem of the spectroscopic orbit.

[1] *Preliminary General Catalogue of 6188 Stars.*

[2] *Popular Astronomy*, **18**, 485, 1910.

[3] *Groningen Publications*, **24**.

[4] *Astrophysical Journal*, **53**, 51, 1921.

Cape observations point to a + sign for the inclination; a series of plates taken during the next apparition, when the ascending node is being closely approached, will remove any doubts.

Assuming the sign positive we have the following values for V_0 based on unpublished details of my measures made from 1908 to 1913, viz.:

Mean Date	Number of Plates	Radial Velocity	Correction for Orbital Velocity	V_0
		km	km	km
1909.35...	13	−3.74	+0.19	−3.55
1910.73...	7	−3.74	+0.13	−3.61
1911.46...	12	−3.51	+0.09	−3.42
1912.49...	13	−3.56	+0.04	−3.52
Means 1911.03...	(45)	−3.63	+0.11	−3.52±0.057

Adopting −3.5 km/sec. as the value of V_0, 0.″33 as the parallax, $\frac{m}{m+m'}$ as $\frac{0.89}{3.59}$, and the elements of Boss, the value of K becomes 0.60 km/sec. and the following radial velocities are calculated:

Date	$i+$	$i-$	Difference
	km	km	km
1921.0...........	−2.87	−4.13	+1.26
1922.0...........	−2.79	−4.21	+1.42
1923.0...........	−2.75	−4.25	+1.50
1923.5...........	−2.74	−4.26	+1.52 (max.)

It is important to take the opportunity of observing the radial velocities at the approaching node, an opportunity which will not again occur for thirty-nine years, in order to obtain a more accurate value of K when the observations are combined with others made at the node due in 1938. During the next seventeen years the velocity-curve shows the most rapid variations in the radial velocity. It is proposed to take three plates on each of seven nights for the determination of a normal place. Corrected for the solar motion, the system is approaching the sun with a velocity of 19 km/sec. in a path inclined 14° to the line joining sun and star.

JOSEPH LUNT

ROYAL OBSERVATORY
Cape of Good Hope
September 3, 1921